MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Capt. Dudley S. A. Cosby, D.L. for Queen's County, Stradbally Hall, Queen's County, Ireland;

Christopher Clive Langton Gregory, M.A., D.I.C., Chief Assistant, Helwân Observatory, "Melville," Parkstone, Dorset; and

Ernest W. F. Keevil, Broomfield, Abbots Langley, Herts,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Lieut. Gordon Dixon Brown, R.N.R., The Shack, Dolvin Road, Tavistock, S. Devon (proposed by J. S. Fletcher);

Arnold Buxton, M.A., Senior Lecturer, Mathematics Dept., Technical College, Cardiff (proposed by A. E. Conrady);

Herbert Dingle, Lecturer on Astrophysics, Imperial College of Science, S. Kensington, 21 Queen Elizabeth's Walk, Stoke Newington, London, N. 16 (proposed by A. Fowler);

Commander Donald Dowdy, R.D., R.N.R., 1 Cannon Court, Brighton, Sussex (proposed by W. G. Bennett);

Alfred Charles Gardner, M.Inst.C.E., F.G.S., West View, Healing, Lincs. (proposed by R. E. Synge Cooper);

Dr. Henri Nort, Krugerlaan 3, Gouda, Holland (proposed by J. van der Bilt);

Egon Sharpe Pearson, B.A., 7 Well Road, Hampstead, London, N.W. 3 (proposed by H. F. Newall);

Bertrand M. Peek, M.A., Gorse Cliff School, Boscombe, Bournemouth (proposed by T. E. R. Phillips); and
Inkerman Rogers, F.G.S., 42 Balaclava Road, Surbiton, Surrey (proposed by J. Tripplin).

Fifty-four presents were announced as having been received since the last Meeting, including, amongst others :—
Córdoba, Observatorio Nacional Argentino, Catalogue of Zones $-27°$ to $-32°$.
Moscow Observatory, Annals, vol. vi.
Perth Observatory, N.S.W., Astrographic Catalogue, vols. xiii. to xvi.

Mr. Frank Robbins and Dr. W. H. Steavenson were appointed auditors of the Treasurer's accounts for 1921.

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1917.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, 79, 446, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at Kodaikánal Observatory and at Dehra Dûn.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun for the year 1917, and Table II. gives the same particulars for each entire year from 1913 to 1917 inclusive. The areas are given in two forms: firstly, projected areas, *i.e.* as seen and measured on the photographs, these being expressed in millionths of the Sun's apparent disc; secondly, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. gives for each rotation in the year 1917 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots. Table IV. gives the same information for each year as a whole from 1913 to 1917 inclusive. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*; for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1914 on pp. 402 and 403 of vol. lxxvi.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

TABLE I.

No. of Rotation.	Rotation commenced G.M.T. (Civil).		No. of Days on which Sun photographed.	Mean of Daily Areas. Projected.			Corrected for Foreshortening.		
	1917	d		Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
847	Jan.	13·86	27	207	1257	1854	137	837	2217
848	Feb.	10·20	28	312	1960	1689	233	1502	2109
849	Mar.	9·53	27	199	1095	1637	144	813	2085
850	Apr.	5·83	27	250	1474	1584	174	1061	1960
851	May	3·09	28	353	2045	1880	244	1435	2315
852	May	30·31	26	420	2419	2417	305	1782	2935
853	June	26·51	28	381	2210	2679	274	1583	3189
854	July	23·71	27	703	4259	2838	494	3089	3446
855	Aug.	19·94	27	276	1660	2545	214	1332	3070
856	Sept.	16·19	27	344	2333	1724	253	1752	2182
857	Oct.	13·47	28	253	1567	1150	186	1151	1418
858	Nov.	9·77	27	238	1606	931	172	1213	1198
859	Dec.	7·09	28	518	3232	1353	369	2367	1744

TABLE II.

Year.	No. of Days on which Sun photographed.	Mean of Daily Areas. Projected.			Corrected for Foreshortening.		
	d	Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
1913	365	2	10	87	1	7	95
1914	365	40	195	396	31	152	454
1915	365	186	920	1289	137	697	1521
1916*	365	186	978	1512	135	724	1785
1917	365	346	2100	1882	248	1537	2305

TABLE III.

No. of Rotation.	Rotation. commenced G.M.T. (Civil).		No of Days on which Sun photographed.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
				Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1917	d			°		°	°	
847	Jan.	13·86	27	393	17·39	444	16·33	−0·49	16·83
848	Feb.	10·20	28	417	19·83	1085	16·22	−6·21	17·22
849	Mar.	9·53	27	586	14·14	227	13·38	+6·46	13·93
850	Apr.	5·83	27	695	13·70	366	18·41	+2·66	15·29
851	May	3·09	28	606	12·51	829	17·45	−4·79	15·36
852	May	30·31	26	902	14·32	880	15·07	−0·20	14·69
853	June	26·51	28	1130	15·17	453	16·49	+6·11	15·55
854	July	23·71	27	2174	14·18	915	16·92	+4·97	14·99
855	Aug.	19·94	27	583	14·01	749	17·76	−3·86	16·12
856	Sept.	16·19	27	951	15·51	801	14·19	+1·94	14·90
857	Oct.	13·47	28	496	11·35	656	15·85	−4·14	13·91
858	Nov.	9·77	27	512	11·06	701	13·38	−3·06	12·40
859	Dec.	7·09	28	1416	9·46	952	12·03	+0·82	10·49

* These values for the year 1916 supersede those given in *M.N.R.A.S.*, 79, 446, for which the complete material was not then available.

TABLE IV.

Year.	No. of Days on which Sun photographed.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	d		°		°	°	°
1913	365	5	24·81	2·5	20·08	+9·81	23·23
1914	365	99	21·11	53	23·06	+5·71	21·79
1915	365	379	17·95	318	19·75	+0·77	18·77
1916*	365	470	15·14	254	17·05	+3·83	15·81
1917	365	860	13·82	677	15·66	+0·84	14·63

The principal features of the record for the year 1917 are as follows:—

1. The great increase of the spotted area from a mean daily value of 724 (corrected for foreshortening) for the year 1916 to 1537 for the year 1917. It should be remarked, however, that the spot activity for 1916 showed but little increase from that of 1915. The year 1917 is both the maximum year of the present sun-spot cycle and the one of greatest spotted area as yet recorded at Greenwich (*i.e.* since 1874).

2. The results for the separate rotations show, as usual, considerable fluctuations in spot activity, with a well-defined maximum during rotation 854, and a secondary maximum for rotation 859.

3. The variation in the amount of faculæ was relatively less than that for the spotted area. The maximum activity occurred in the middle part of the year, in particular during rotation 854.

4. Comparing the two hemispheres, the preponderance was slightly with the northern spots, 56 per cent. of the spotted area occurring in the northern hemisphere. This hemisphere has been predominant in activity since the last minimum.

5. The mean distance of all spots from the equator has shown the characteristic progression equatorwards since the minimum. The value for 1917 accords closely with that for 1893, which is the maximum year of a cycle similar in several respects to the present one.

6. On no day in the year was the Sun entirely free from spots.

7. The number of separate groups of spots observed during 1917 for at least two days was 455 as compared with 339 for 1916. Of these 455 groups, 249 were in the northern hemisphere and 206 in the southern. Besides these groups, a large number of small spots were seen for one day only; of the 202 recorded during the year, 126 were in the northern hemisphere and 76 in the southern. Most of these spots were very small, but their reality was always confirmed on at least one other photograph, sometimes with a time interval of one or two hours. An unusual number of these small spots has been already noted during 1915 and 1916.

Royal Observatory, Greenwich:
1921 *November.*

* These values for the year 1916 supersede those given in *M.N.R.A.S.*, 79, 446.

Effective Wave-lengths of Stars in relation to Spectral Type and Colour Index. By C. Davidson and E. Martin.

(*Communicated by the Astronomer Royal.*)

When a coarse grating is placed before an objective, short diffraction spectra are produced in the focus on either side of a central image, the distance between the diffracted and central images being a function of the grating interval, the focal length of the objective, and the wave-length of the light employed.

When applied to stellar photography this furnishes a convenient means for determining the colours of the stars as defined by the wave-length of maximum photographic intensity.

The method was employed by the brothers Henry for the determination of the influence of atmospheric dispersion on the observations of Eros for the determination of the Solar Parallax,* and was later applied by Hertzsprung and Bergstrand to the determination of the colours of stars.

If a is the width of the bar,
b the width of the clear space,
f the focal length of the objective,
D the measured distance between the first-order spectra,
and x, y the co-ordinates of the star on the plate,
then

$$\lambda = \mathrm{D} \cdot \frac{a+b}{2f} \cdot \left\{ 1 - \frac{3x^2+y^2}{2f^2} \right\}.\dagger$$

At the greatest distance at which measures were made (*i.e.* $x = 65'$, $y = 65'$) the correction involved in the second term only amounts to 2·3 A, and has therefore been neglected.

If B is the brightness of the image without the grating and B_m that of the m^{th} lateral image, we have

$$\mathrm{B}_0 = \mathrm{B} \cdot \frac{a^2}{(a+b)^2}.$$

$$\mathrm{B}_m = \mathrm{B} \cdot \frac{1}{m^2\pi^2} \cdot \sin^2 \frac{am\pi}{a+b}.$$

When bar and space are equal these become

$$\mathrm{B}_0 = \tfrac{1}{4} \cdot \mathrm{B}$$

$$\mathrm{B}_m = \mathrm{B} \cdot \frac{1}{m^2\pi^2} \cdot \sin^2 \tfrac{1}{2}m\pi.$$

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* Conférence Astrographique, *Circular* 8, 1901.
† Lindblad, *Arkiv för Mathematik, etc.*, Stockholm, Band 13.

In this case the first-order spectrum contains the utmost amount of light obtainable in any one spectrum, and

$$B_1 = \tfrac{1}{10}\, B,$$
$$B_2 \quad \text{disappears},$$
$$B_3 = \tfrac{1}{9}\, B_1.$$

As the third-order spectrum is three times the length of the first order, the intrinsic brightness of B_3 is $\frac{1}{27}^{\text{th}}$ of that of B_1.

The work which follows was undertaken with a view to testing the method in its general application to the determination of the effective wave-lengths of stars for the purpose of supplying an indication of spectral type and colour index for those stars in the Greenwich section of the Astrographic Catalogue not contained in the Draper Catalogue.

The telescope employed was the 30-inch reflector, focal length 3476·0 mm., stopped down to 20 inches to increase the field of useful definition. As a commencement, work was confined to the region within 5 degrees of the Pole.

The grating, of equal bar and space, was at Professor Hertzsprung's suggestion made of wire approximately 1·5 mm. diameter spaced at 3 mm. The actual dimensions were 1·423 mm. and 2·846 mm. The distance between the first-order images on the photograph is a little more than 1 mm.

The photographs were taken on plates 16 cm. square, covering a field of 2°, the scale being the same as that of the Astrographic telescope, viz. 1 mm. = 1 minute of arc. Although the lateral images are much affected by optical distortion, this does not greatly depreciate their value in this special work. Both spectra being equally affected, it is found that nearly as reliable measures can be made on the diffused images as on the better defined.

Usually two exposures of 10^m and two of 2^m duration were given on each plate. On the later plates, to make further provision for the very bright stars two additional exposures of 24^s each were added. The longest exposure shows measurable images down to the 11^{th} magnitude, the plates being measured down to the $10\frac{1}{2}$ magnitude.

The plates were measured with a filar micrometer by each of two measurers, and to avoid any change of system the same measurers continued throughout. In addition to the first-order spectra, the third orders also were measured when sufficiently well shown.

Effect of Exposure.—The range of exposure was given, partly to determine and partly to avoid the troublesome correction necessary to reduce all measures to a common standard magnitude due to the fact that the short spectra do not increase uniformly in both directions with an increase of exposure.

For this purpose all the measured wave-lengths as given by the 10^m and 2^m images were compared, and arranged in order of wave-length and again in order of magnitude and formed into 9 groups. These gave equations of the form

$$a\lambda + bm + c\lambda^2 + dm^2 + e\lambda m + f = \lambda(10^m) - \lambda(2^m).$$

In the course of the work two makes of plates had been used, Ilford Special Rapid and Seed Gilt Edge. In this discussion they were treated separately, and the following solutions obtained :—

	Special Rapid.	Gilt Edge.
a	+ ·055	+ ·057
b	+ 9·80	+ 7·60
c	− ·00090	− ·00040
d	− 7·48	− 5·99
e	+ ·0374	− ·0170
f	+20·6	+13·6

e being discordant was considered unreal, and after giving it a value 0 and re-solving the equations a mean value

a	+ ·070
b	+ 9·00
c	− ·00067
d	− 6·75
f	+17·4

was adopted.

This expression represents the increase in wave-length produced by an increase of intensity equal to 1·5 magnitudes.*

From these figures the following formula was derived for the correction for stars of all colours, to reduce to an image of standard density:

$$\text{Corrn.} = (m-10)\{\cdot047(\lambda-4300) - 2\cdot62(m-10) - \cdot00045(\lambda-4300)^2 - 1\cdot50(m-10)^2 + 12\cdot90\}.$$

The following table was then constructed, giving the correction to the measured wave-lengths to reduce to that given by images of standard density. The standard density adopted is that given by a 10 minutes exposure on a 10th magnitude star, an image neither under nor over exposed, but where the maximum intensity is easily measurable.

As the constants were derived from a comparison of images differing by 1·5 magnitudes only, a correction for more than that amount is considered an extrapolation, and weighted accordingly. Thus for stars fainter than $8^{m}\cdot5$ the 10-minute and 2-minute images are weighted equally in the mean. From $7^{m}\cdot0$ to $8^{m}\cdot5$ the 10 minutes is given half weight. Brighter than $7^{m}\cdot0$ the 10 minutes is not used, but the third-order image of intensity $10^{m}\cdot5$ becomes measurable, and is combined with the 2-minute images.

The increase in intensity of the red with increased exposure is analogous to the Purkinje effect, and is sometimes described as such. It will be noticed that the action is not continuous. Some light may be derived from a paper by F. E. Ross † who shows that the *gamma* or

* When a value ·85 is given to p in the equation—
photographic effect $= It^{p}$.

† *Astrophysical Journal*, 52, 2.

growth of density with exposure is not the same for all wave-lengths, being high at λ4800 and low at λ4200. It would appear that the red end at first gains density more rapidly than the blue, with a consequent increase in effective wave-length. With a continued exposure, however, the growth of the red is checked by the insensitiveness of the plate in that region, whilst the blue continues to gain in density, so readjusting the balance.

TABLE I.

Correction to reduce Measured Wave-length to Standard Intensity.

(Unit = 1 A.)

λ / Mag.	4200.	4250.	4300.	4350.	4400.	4450.	4500.	4550.
7·0	+6	−11	−22	−25	−22	−12	+4	+26
7·5	−2	−16	−25	−28	−26	−17	−4	+15
8·0	−6	−17	−24	−27	−25	−18	−7	+ 8
8·5	−6	−15	−20	−22	−20	−15	−7	+ 4
9·0	−5	−10	−14	−15	−14	−11	−5	+ 2
9·5	−2	− 5	− 7	− 8	− 7	− 5	−3	+ 1
10·0	0	0	0	0	0	0	0	0
10·5	+1	+ 4	+ 6	+ 6	+ 6	+ 4	+1	− 2
11·0	0	+ 5	+ 9	+10	+ 9	+ 6	0	− 7

Atmospheric absorption, which at sea-level and at Z.D. 60° will cause an increase in effective wave-length amounting to 35 A, appears in two classes of error—that due to the photographs being taken at different zenith distances, and that due to varying transparency from night to night. The first has been avoided by taking all the photographs at the altitude of the Pole; the second has been checked by exposing at least one plate on the polar region whenever photographs were taken, from which a criterion as to the nightly absorption might be inferred. This is analogous to the procedure in photographic photometry of photographing the Polar Sequence on each plate. From the results obtained this would appear to be important, as the wave-lengths obtained on different nights may differ by as much as 30 A. This, it should be noted, is extreme, and appears on plates taken on nights the observer has noted as hazy. Hertzsprung also finds a systematic error common to all wave-lengths derived from one plate estimated at ± 10 A.* Our results indicate that the night error is of the greater importance.

From a comparison of residuals we obtain a mean discordance of ± 20 A, or as probable error of ± 17 A, for a single night. The probable error of the results printed, each of which is derived for three or more plates, may be taken as ± 10 A.

The results for those stars whose spectral type is known are given in the following table, which is arranged in order of B.D. zone and number. The photographic magnitudes given in column 3 are derived from photographs taken with a 6-inch lens, and corrected to the Harvard system (*H.A.*, 71, 3).† The colour index is the difference between this

* *Astrophysical Journal*, 42, 1.

† Photographic magnitudes of stars brighter than $9^{m}\cdot0$ between Dec. +75° and the Pole, determined at the Royal Observatory, Greenwich, 1913.

photographic and the Harvard photometric magnitudes. The spectral type is taken from the Draper Catalogue, and in the last column is found the effective wave-length of maximum photographic intensity as determined from the photographs. As only a few of the very early-type stars occur in the limited region under observation, a few selected B stars were observed in other parts of the sky; these are given at the end of the table.

TABLE II.

Effective Wave-lengths of Stars near the North Pole.

B.D. No.	R.A. 1900·0. h m	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.
89° 3	3 19	9·03	·18	A_5	4267
13	7 58	7·16	·15	A_0	4262
21	12 42	9·46	·46	F	4284
22	12 52	10·10	1·43	K_0	4512
88 2	0 16	8·82	·72	K_0	4395
4	0 56	6·55	·07	A_2	4268
8	1 23	2·69	·57	F_8	4380
9	2 14	8·25	·24	F_0	4259
11	2 17	8·82	·42	G_0	4259
13	2 42	8·82	·21	A	4265
39	7 17	9·15	·24	F_0	4264
60	10 19	8·99	·25	F_8	4297
64	11 4	7·32	−·12	B_8	4244
71	12 14	6·59	·31	F_0	4278
76	13 4	8·78	1·21	K_2	4513
77	13 27	8·69	·24	F_2	4250
80	13 54	8·99	·07	A_0	4254
86	14 18	8·71	−·07	B_9	4237
90	15 5	9·63	·66	K_0	4380
104	17 54	8·33	·22	F_5	4268
105	52	8·96	·76	K_0	4363
110	18 49	9·13	·37	F_5	4246
112	19 22	7·82	1·27	Mb	4519
114	19 43	9·09	·83	K_0	4424
115	19 54	9·03	·15	A_2	4245
117	19 59	8·89	var. (·54)	G_5	4304
130	21 57	9·35	·24	F	4275
131	22 12	8·83	·23	A_5	4263
87 1	0 7	8·98	...	A	4256
9	1 0	8·90	...	F_5	4278
12	1 18	8·91	·87	K_0	4472
15	1 50	8·12	·38	A_3	4264
16	1 56	9·07	·43	G_0	4315
87° 23	2 52	9·29	·86	K_0	4431
33	4 5	8·66	·24	F_2	4258
41	5 46	9·87	1·23	K_0	4567
46	6 35	9·08	...	G_5	4425
51	6 54	6·66	1·40	Ma	4549
68	8 26	8·87	...	G_0	4264
78	9 18	8·34	·07	F_5	4273
79	9 28	9·24	...	K_0	4451
80	9 29	9·55	...	K_0	4478
83	9 44	8·63	·66	G_5	4414
85	10 4	9·09	...	K_0	4412
99	11 47	8·78	...	G_5	4335
101	11 55	7·90	·14	A_2	4262
104	12 8	8·98	...	K_5	4471
107	12 14	6·65	·32	F_2	4286
115	12 58	8·86	...	F_8	4273
118	13 14	9·71	...	K_0	4456
121	13 20	9·55	...	G_5	4441
122	13 25	8·52	...	F_2	4266
143	15 9	8·18	1·02	K_0	4528
147	15 27	8·37	·30	F_8	4286
151	16 5	9·24	...	F_8	4273
169	18 4	8·35	...	F_0	4271
180	19 15	8·38	·08	A_5	4252
181	19 16	8·48	·14	A_3	4246
187	20 25	9·06	1·02	K_2	4475
201	21 59	8·53	...	F_8	4270
205	22 24	7·38	·01	A_2	4258
217	23 43	8·84	...	A_0	4246
220	0 0	9·69	...	G_5	4432
86 9	0 37	8·57	...	A_3	4260
14	49	9·23	...	G_5	4348
17	59	7·15	·75	K_0	4469

TABLE II.—*continued.*

B.D. No.	R.A. 1900·0.	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.	B.D. No.	R.A. 1900·0.	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.
	h m						h m				
86° 21	1 11	8·81	...	G_0	4318	86° 282	18 48	8·00	1·18	Ma	4532
25	40	8·15	...	F_5	4288	318	21 16	8·61	·27	F_2	4268
38	2 25	9·08	...	A_5	4279	319	20	7·34	−·02	A_3	4268
39	32	7·91	...	F_0	4264	324	46	8·31	...	A_2	4267
51	3 34	6·13	·29	F_5	4287	325	47	9·86	...	K_0	4363
54	53	8·82	...	F_0	4269		48	10·25			4391
65	4 38	8·81	...	G_5	4314	326	48	9 96	...	K_5	4548
66	46	7·78	−·18	B_9	4233	332	22 22	9·24	...	F_2	4255
67	51	9·16	...	G	4284	335	42	8·00	−·07	A_0	4244
79	6 8	7·64	1·07	G_5	4486	344	23 28	5·82	·20	F_0	4277
86	23	9·12	...	F_8	4268	347	57	7·86	...	F_0	4270
91	35	9·28	...	A	4279						
96	50	9·05	...	G_5	4291	85 2	0 9	8·67	...	G_0	4325
102	7 10	8·62	...	A_0	4266	3	9	9·81	...	K_0	4549
103	11	8·24	...	G_0	4304	9	26	8·43	...	K_0	4384
110	47	9·72	...	Ma	4524	11	29	8·25	...	A_2	4278
113	7 52	8·13	·79	G_5	4423	19	55	5·55	1·03	K_0	4551
126	8 47	9·12	·88	K_0	4533	41	1 59	7·64	·40	G_5	4420
143	9 58	8·27	...	A_3	4276	45	2 23	7·61	...	A_0	4265
146	10 2	9·08	...	A_0	4275	50	41	8·44	...	A_0	4256
154	41	8·38	...	G_0	4310	63	4 5	6·99	·29	F_8	4329
159	57	8·34	...	A_3	4284	65	21	8·56	...	F_5	4297
161	11 2	7·39	·22	A_2	4274	74	56	6·69	·15	A_5	4306
170	28	7·51	·14	F_0	4257	75	5 0	8·75	...	G_5	4426
172	40	8·74	...	K_0	4410	78	10	6·55	·00	A_0	4250
176	12 0	6·75	·37	F_5	4328	80	30	7·59	1·18	K_0	4561
177	12 7	8·89	...	G	4253	81	35	8·37	·76	G_5	4462
180	20	8·79	...	A_0	4274	98	6 36	9·62	...	Ma	4548
182	35	7·31	·24	F_0	4270	105	51	8·25	...	F_8	4306
187	13 0	8·00	·25	F_8	4294	132	48	8·47	...	F_0	4295
193	32	8·09	·05	F_2	4278	142	9 9	8·89	...	G_0	4310
199	55	9·18	...	F_5	4277	147	20	9·63	...	K_5	4567
201	59	7·48	·04	F_0	4285	155	10 4	9·36	·80	G_5	4426
217	14 50	7·84	·82	K_0	4446	160	20	8·69	...	G_0	4296
221	15 6	8·01	...	F_8	4281	166	31	9·17	...	K_0	4477
244	16 35	var.	...	A_0	4283	170	47	9·21	·53	G_5	4437
256	17 12	9·10	...	G_5	4453	183	11 24	8·06	·71	K_0	4405
264	33	8·48	·44	F_2	4261	191	45	8·74	...	F_5	4278
269	18 5	4·48	·04	A_0	4255	196	12 7	8·61	...	A_3	4278
272	8	5·96	·10	A_3	4272	214	13 1	9·56	·38	G_5	4311
275	26	8·59	...	K_0	4461	222	13 19	7·58	·23	G_0	4300

TABLE II.—*continued.*

B.D. No.	R.A. 1900·0.	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.	B.D. No.	R.A. 1900·0.	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.
°	h m					°	h m				
233	13 43	9·09	...	G	4297	84 18	1 3	8·95	...	F_5	4289
234	51	8·82	·87	K_0	4493	29	34	8·03	·02	A_5	4281
235	56	9·66	...	K	4534	30	35	8·79	...	A_5	4278
242	14 38	9·62	1·04	K_0	4505	32	41	9·32	...	G_5	4443
249	15 9	8·21	·80	K_0	4445	34	46	8·72	...	G_5	4470
263	43	7·60	·62	K_0	4423	42	2 20	8·94	...	G_5	4349
264	43	9·38	·45	K	4338	48	45	8·74	...	A	4286
266	50	8·92	...	G	4289	59	3 9	6·57	·79	K_0	4453
269	57	6·91	−·14	A_5	4263	78	4 9	8·53	1·12	K_2	4541
278	16 50	8·75	...	F_8	4299	93	48	9·16	·68	K_0	4546
294	18 7	7·63	·13	F_0	4288	97	4 58	8·76	...	F_5	4292
329	19 34	8·89	...	K_0	4470	112	5 44	9·05	·27	G	4310
330	36	9·26	...	G_5	4363	225	9 53	7·82	1·34	K_0	4571
337	20 3	8·74	...	A_5	4257	234	10 15	5·92	·28	A_3	4310
340	14	9·15	1·20	K_2	4557	303	13 11	9·43	·25	A	4268
347	27	9·12	·57	G_5	4364	335	15 2	8·20	1·08	K_0	4516
354	50	9·16	1·25	K_0	4550	339	7	9·46	...	K_0	4491
355	52	9·38	...	G_5	4261	351	16 1	7·27	·21	A_3	4304
357	59	9·39	·41	F_8	4312	352	0	8·15	·12	F_8	4274
359	21 7	8·37	...	F_0	4265	354	5	9·02	...	F_8	4236
367	51	8·75	...	A	4259	370	57	9·21	...	K_2	4452
376	22 2	8·83	...	F_0	4271	371	59	9·28	1·05	K_5	4561
383	21	5·30	−·08	A_0	4239	378	17 12	7·70	−·03	A_0	4278
384	22	7·59	·81	K_0	4488	389	44	8·67	...	A_3	4251
390	43	9·15	...	G_5	4385	391	49	8·88	...	G_5	4334
398	23 19	9·41	...	K_2	4467	405	18 7	9·08	...	G_5	4454
399	24	6·75	·14	F_0	4293	410	21	8·50	...	F_0	4282
400	26	8·20	·44	G_5	4391	412	25	7·43	·05	F_2	4276
401	28	7·45	·20	F_5	4275	413	32	8·21	...	A_5	4276
403	30	7·07	−·11	A_5	4258	423	47	8·71	...	F_0	4265
406	51	8·57	−·11	A_0	4252	426	19 1	8·99	...	F_8	4273
409	23 55	6·70	−·01	A_0	4283	436	21	9·18	...	F_5	4280
412	0 2	8·66	...	F_0	4282	437	26	10·0	...	K_2	4560
						445	54	8·06	...	A_0	4256
3	0 10	7·69	−·22	A_2	4280	446	57	8·83	...	G_0	4330
4	12	8·99	...	A	4259	448	20 5	8·88	...	A_0	4272
9	27	8·71	...	G_0	4291	451	14	6·82	·21	A_2	4282
10	29	8·64	...	K_0	4430	461	23	8·62	...	K_0	4453
13	38	8·61	...	A_2	4259	462	24	7·42	·26	F_0	4288
14	44	8·86	·53	K_0	4454	463	25	7·46	·35	F_8	4321
15	45	8·11	...	A_5	4276	474	53	8·36	...	A_2	4264

TABLE II.—*continued.*

B.D. No.	R.A. 1900·0. h m	Photog. Mag.	Colour Index.	Spectral Type.	Eff. λ.
84° 505	22 14	8·56	·51	G_0	4360
509	27	7·93	·72	K_0	4436
513	50	7·85	·76	K_0	4471
516	53	8·57	·97	Mb	4431
517	53	7·12	·94	K_5	4545
523	23 7	8·72	·34	F	4297
533	34	8·33	...	A_2	4272
536	39	8·41	...	K_0	4446
539	44	8·39	...	G_5	4400
546	0 1	8·12	...	A_3	4273
83 1	0 4	8·30	...	F	4271
9	30	7·46	·36	F_8	4320
10	31	8·24	...	F_8	4323
11	31	8·55	...	F_5	4301
20	53	6·80	·08	F_5p	4295
30	1 14	9·11	...	A_0	4256
52	2 6	9·15	...	A_2	4277
527	18 9	7·90	·31	G_5	4362
533	28	9·04	...	A_5	4255
552	19 28	6·72	·38	A_2	4282
588	20 39	6·48	·32	A_2	4283
594	54	9·23	...	K_0	4478
596	59	7·50	·28	F_2	4289
603	21 22	7·96	·90	G_5	4493
608	27	8·97	...	F_5	4266
613	35	8 58	...	F_0	4277
614	40	8·24	...	F_5	4302
616	46	8·17	...	A_3	4268
617	47	9·08	...	K_0	4402
618	50	7·41	·39	A_5	4275
620	55	9·25	...	K_0	4474
622	22 4	8·90	...	A_5	4276
630	21	7·35	− ·14	A_0	4249
640	55	6·13	1·17	K_0	4570
647	23 13	7·55	− ·21	B_9	4219
82 14	0 28	7·89	...	A_2	4277
20	45	5·93	·38	A_2	4282
82° 608	20 10	8·35	...	A_5	4280
617	34	7·57	·75	G_5	4455
650	21 30	8·89	...	G_0	4286
651	31	8·65	...	F_8	4295
672	22 1	7·85	...	F_0	4283
682	13	8·90	...	K_0	4455
689	22	8·91	...	G_5	4318
700	44	7·37	− ·09	B_8	4252
703	48	6·06	1·09	K_0	4522
704	49	8·25	·69	K_2	4404
707	57	9·26	...	K_0	4450
720	23 24	9·21	...	F_5	4287
83 297	10 19	5·63	+ ·29	F_2	4277
38 2179	10 43	6·6	...	B_3	4019
69 602	11 6	6·44	+ ·02	A_2	4245
69 644	12 1	7·25	+ ·11	F_5	4230
κ Draco.	12 29	3·72	− ·16	B_5p	4157
5 2737	13 17	5·9	...	A_0	4268
η Urs. Maj.	13 44	1·7	...	B_3	4144
66 878	14 56	6·16	+1·30	Mb	4487
μ Bootis	15 21	4·5	...	F_0	4280
θ Coronæ	15 29	4·0	...	B_5	4180
ζ Draco.	17 9	3·14	− ·08	B_5	4181
36 3243	18 39	9·3	...	N	4572
δ^1 Lyræ	18 50	5·3	...	B_3	4145
δ^2 Lyræ	18 51	5·6	...	Mb	4527
19 Vulpec.	20 8	5·8	...	K_3	4531
26 3826	20 8	7·0	...	B	4167
26 3827	20 8	7·4	...	B_5	4179
20 Vulpec.	20 8	6·8	...	B_8	4215
25 4165	20 11	4·6	...	B_3	4175
23 Vulpec.	20 12	5·7	...	K	4486
15 4220	20 34	5·8	...	B_2	4180
α Delphini	20 35	3·9	...	B_8	4256
42 4220	22 26	4·2	...	B_3	4135
38 4826	22 35	4·6	...	Oe_5	4074
39 4912	22 37	4·9	...	B_2	4115

In order to examine the relationship between effective wave-length and spectral type, the wave-lengths have been grouped in their spectral classes as shown in Table III. The mean for each type with the range and mean discordance is given in Table IV. The results of Lindblad * and Max Wolf † are also shown for comparison.

* *Arkiv för Mathematik, etc.*, Band 13, No. 26.
† *Ast. Nach.*, 5092.

TABLE III.

Wave-lengths grouped in Spectral Classes.

Oe_5.	B_5.	A_0.	A_2.	A_3.	A_5.	F_0.	F_2.	F_5.	F_8.	G_0.	G_5.		K_0.			K_2.	K_5.	Ma.
4074	4157	4239	4245	4246	4252	4257	4250	4230	4236	4259	4261	4437	4363	4455	4551	4404	4471	4524
	179	244	258	251	255	259	255	246	268	264	291	441	363	456	561	452	545	532
	180	246	259	260	257	264	258	266	270	269	304	443	380	461	567	467	548	548
	181	249	262	264	258	264	261	268	273	286	311	453	384	469	570	475	561	549
		250	264	268	263	265	266	273	273	291	314	454	391	470	571	513	567	
B_2	B_8	252	267	268	263	265	268	275	273	296	318	455	395	471		541		
		254	268	272	267	269	276	277	274	300	334	462	402	472		557		Mb
4115	4215	255	272	273	275	270	277	278	281	304	335	470	405	474		560		4431
180	244	256	274	276	276	270	278	278	286	310	348	486	410	477				487
	252	256	277	278	276	271	286	280	294	310	349	493	412	478				519
	256	256	277	284	276	271	289	287	295	315	362		423	478				527
		262	278	304	278	277		287	297	318	363		424	488				
		265	280	310	279	277		288	299	325	364		430	491				
B_3	B_9	266	282		280	278		289	306	330	385		431	493				
		268	282		281	280		292	312	360	391		436	505				N
(4019)	4219	272	282		306	282		295	320		400		445	512				4572
135	233	274	283			282		297	321		414		446	516				
144	237	275				283		301	323		420		446	522				
145		278				285		302	329		423		450	528				
175		283				288		328	380		425		451	531				
		283				288					426		453	533				
						293					426		453	546				
						295					432		454	549				
														550				

TABLE IV.

Comparison of Effective Wave-length with Spectral Type.

Type.	Effective Wave-length.	No. of Stars.	Range of Wave-length.	Mean Discordance.	Lindblad.		Max Wolf.	
					Effective Wave-length.	No. of Stars.	Effective Wave-length.	No. of Stars.
Oe_5	4074	1	...	...	4123	2		
B_2	4148	2	65	±22	4118	6		
B_3	4150	4	40	13	...	...	4190	2
B_5	4174	4	24	9	4112	4		
B_8	4242	4	41	13	4159	5	4230	1
B_9	4230	3	18	7				
A_0	4261	20	44	11	4179	9		
A_2	4271	17	38	9	4192	6	4310	2
A_3	4273	13	64	13				
A_5	4271	16	54	11	...	...	4320	1
F_0	4275	22	38	9	4219	4	4330	2
F_2	4269	10	39	11				
F_5	4285	19	82	13	4236	5	4370	1
F_8	4296	20	93	23				
G_0	4306	14	96	18	4281	3	4420	3
G_5	4394	33	232	51	4296	4	4480	3
K_0	4468	51	208	45	4326	9	4540	3
K_2	4496	8	156	46				
K_5	4538	5	96	27	4420	3	4570	2
Ma	4538	4	25	10	4434	4	4570	3
Mb	4491	4	96	30	...	...	4530	3
N	4572	1	...	...	...	...	4710	2

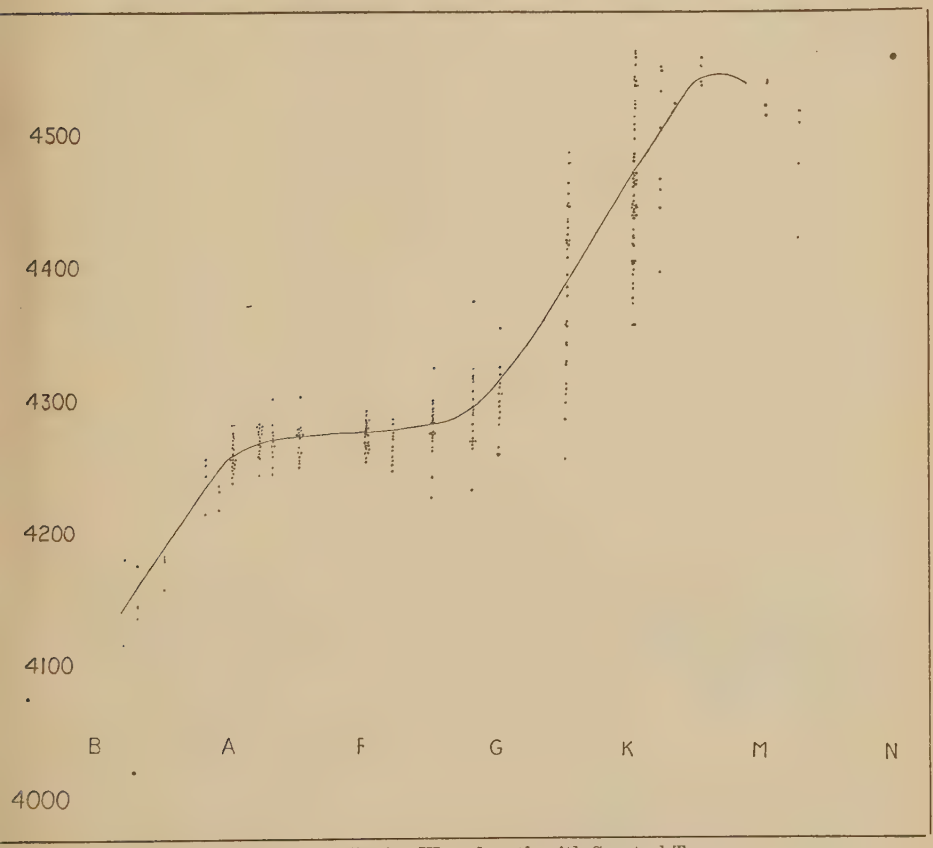
The results given in Table III. are exhibited graphically in the diagram. The smoothed curve very closely follows the means given in Table IV.

It will be observed that the progression of wave-length against spectral type is not linear. From B_0 to A_0 the gradient is steep. Here there is a pause, and from A_0 to G_0 the increase in wave-length is very slight. From G_0 to K_5 there is another sharp gradient, with another pause at M. The halt from A_0 to G_0 should be remarked, depending as it does on the observations of a large number of stars. Although not entirely supported by the other observers, we believe it to be real. The rapid rise from B to A_0 is probably caused by the strength of the hydrogen absorption lines, which at A_0 reach their maximum intensity. The fading out of these, which at F_0 are of intensity ·4, might lead to a partial recovery in the blue.

The spreading of the wave-lengths at G_5 and K is a point of great interest. It may be due to—

(1) Error in measured wave-length.
(2) Error in spectral classification.
(3) Peculiarities in the spectra.

As regards (1): reference to Table IV. shows that in G_5, K_0, K_2 the mean discordance is more than four times the probable error; and



Comparison of Effective Wave-length with Spectral Type.

further, it is precisely this type with shortened spectra which are most easily measured.

As regards (2): we have no evidence, but if it is permissible to assume a possibility of error of half a type amongst the fainter stars, much of the scattering can be accounted for.

As regards (3): dwarf stars are likely to appear blue in their class, and giants red. Amongst these stars we have several with measured parallaxes. These are given in the following table, from which it will be seen that the giants are distinctly red and the dwarfs blue, in their classes.

Comparison of Wave-length, Spectral Type, and Absolute Magnitude.

Star.	π.	Abs. Mag.	Type.	Wave-length.		
				Measured.	From Curve.	Diff.
	″	m				
88° 8	·010	−2·3	F_8	4380	4300	+80
85 19	·016	0·5	K_0	4551	4480	+71
85 80	·008	0·7	K_0	4561	4480	+81
83 640	·012	0·4	K_0	4570	4480	+90
87 51	·011	0·4	Ma	4549	453	
δ^2 Lyræ	·007	−1·2	Mb	4527	451	
19 Vulpec.	·009	0·5	K_3	4531	4530	+ 1
66 878	·008	−0·6	Mb	4487	451	
μ Boötis	·035	2·2	F_0	4280	4280	0
87 107	·031	3·9	F_2	4286	4282	+ 4
88 71	·014	2·0	F_0	4278	4280	− 2
83 297	·025	2·3	F_2	4277	4282	− 5
86 14	·004	1·8	G_5	4348	4395	−47
87 99	·026	5·7	G_5	4335	4395	−60
82 704	·031	5·0	K_2	4404	4500	−96

The evidence is not strong, but so far as it goes supports this hypothesis.

Comparison of Wave-length and Colour Index. — To examine the relationship between wave-length and colour index, the stars were arranged in order of wave-length and divided into groups, the means of which are given in the following table:—

Wave-length.	Colour Index.	No. of Stars.
4240	− ·02	10
4261	+ ·14	27
4280	+ ·19	28
4308	+ ·28	17
4400	+ ·65	21
4484	+ ·93	20
4550	+1·11	15

and from these was derived

$$\lambda_e = 4226 + 280\,\mathrm{I}.$$

The colour indices of the stars of the North Polar Sequence have been determined both at Mount Wilson and Harvard. These are given in the following list, together with the measured wave-lengths:—

No.	B.D. No. °	Photog. Mag. M.W.	Colour Index. M.W.	Colour Index. H.	Effective Wave-length.	No.	B.D. No. °	Photog. Mag. M.W.	Colour Index. M.W.	Colour Index. H.	Effective Wave-length
1	86 269	4·14	+·02	+·04	4255	1*s*	88 8	2·29	+·46	+·57	4380
2	85 383	5·05	+·02	−·10	4239	2*s*	88 71	6·20	·12	·20	4228
3	86 344	5·58	+·27	+·19	4277	3*s*	87 107	6·39	·29	·32	4286
4	86 272	5·66	+·07	+·13	4272	4*s*	89 12	10·00	·42	·54	4304
5	88 4	6·20	−·02	+·06	4268	5*s*	89 37	10·84	1·03	·95	4436
6	89 13	6·86	+·06	+·10	4262	6*s*	89 26	11·13	·65	·36	4367
7	88 64	7·16	−·11	−·13	4244						
8	88 9	8·07	+·19	+·26	4259	1*r*	87 51	6·36	1·52	1·54	4549
9	88 13	8·63	·07	·22	4265	2*r*	88 112	7·69	1·59	1·32	4519
10	89 3	8·87	·05	·08	4267	3*r*	88 76	8·71	1·40	1·18	4513
11	89 18	9·48	·20	·25	4267	4*r*	88 114	8·97	·96	·84	4424
12	89 25	9·84	·29	·32	4290	5*r*	89 22	9·88	1·48	1·24	4511
13	89 29	10·28	·16	·05	4278	6*r*	89 9	10·21	1·25	1·05	4477
14	89 1	10·73	·44	·21	4272	7*r*	89 35	10·69	1·04	·86	4457
15	N.P.C. 452	10·97	·33	·30	4268	8*r*	89 31	11·19	1·00	·81	4412
16	N.P.C. 1678	11·37	·39	·33	4257	*a*	N.P.C. 1657	10·76	·37	...	4264

These also were divided into groups in order of wave-length and the following equations derived:

$$\lambda e = 4230 + 200\ \mathrm{I}\ \text{(Mount Wilson)}$$

and

$$\lambda e = 4230 + 230\ \mathrm{I}\ \text{(Harvard)}.$$

The difference in the coefficients of I is the colour equation between the Mount Wilson, Harvard, and Greenwich-Harvard scales. The large coefficient of I in the Greenwich-Harvard result indicates that there is a colour equation to the Greenwich photographic magnitudes determined with the 6-inch lens, of about the same order as that found by Professor Seares in the photographic magnitude determined with the 26-inch refractor.

In conclusion, it seems clear that by means of a grating one can, with suitable precautions, obtain accurately the position of the maximum photographic intensity of the spectra of all stars to a low magnitude. It has only been carried out here to about 10·5 mag., because a survey of the whole of the Greenwich section of the Astrographic Catalogue was in contemplation.

A reflector seems most suitable for the work. The limited field which would militate against its use for many purposes is of little consequence, whilst its freedom from chromatic aberration and selective absorption is of prime importance.

Effective wave-lengths provide a classification of spectra according to the position of maximum photographic intensity, and in conjunction with other spectral classification, as has been pointed out by several astronomers, may give a criterion as to the absolute luminosity of stars.

Note on the Accidental Errors of Argelander's Method.

By P. M. Ryves.

The day after its discovery by Denning, observations of Nova Cygni (No. 3) were commenced by the writer and were continued as long as the star remained visible to the naked eye.

The method of Argelander was employed, but the number of comparisons constituting an observation was somewhat greater than is usually the case.

When a series had been obtained the observations were reduced, taking magnitudes for the comparison stars from Harvard Photometry (H.P.), vol. xiv. of *Harvard Annals*. The agreement between the observations and the catalogue was good, but some small differences were thought to be due to the catalogue rather than to the observations. Values for the magnitudes of the comparison stars were then taken from the Revised Harvard Photometry (H.R.) in vol. l. of *Harvard Annals*, and also from the Potsdam Photometric Durchmusterung (P.D.), and compared with those from the older Harvard catalogue. The result of this comparison is shown in Table I.

The first column contains the designations of those comparison stars for which we have three or more comparisons on not less than two nights. The mean residuals given in the third column may be considered as corrections which the observations suggest should be applied to the H.P. magnitudes, whilst the differences Harvard Photometry minus Revised Harvard Photometry represent the corrections actually applied at Harvard. The differences from Potsdam were obtained after the magnitudes from the latter had been adjusted to the colour scale of H.R.

It will be seen that the residuals H.P. – O are, on the whole, supported by the later photometric measures.

Tables I. and II.		Table I.			Table II.	
1. Comparison Stars.	2. Nights.	3. H.P. – O.	4. H.P. – H.R.	5. H.P. – P.D.	3. H.R. – O.	4. H.R. – P.D.
β Cephei (var.)	3	+06	+07	+17	+07	+10
23 Fl. Cygni	6	+09	+05	+08	±00	+03
δ Cygni	5	+06	+08	+13	−02	+05
33 Fl. Cygni	3	+09	+09	+11	−02	+02
α Cephei	4	−02	−02	+04	+06	+06
ι Cygni	3	−03	−05	+18	±00	+21
γ Cygni	3	−05	−01	−04	−04	−03
χ Cephei	4	+01	−07	+05	+05	+12
η Cephei	3	+07	±00	−03	+06	−03
δ Draconis	3	±00	−03	−09	+05	−06
20 Fl. Cygni (var.)	6	−10	−13	−06	±00	+07
κ Cygni	2	−08	−04	+01	−07	+05
26 Fl. Cygni	6	−02	−08	+10	+03	+18
γ Draconis	2	−06	−07	−15	+05	−08

The first star on the list is slightly variable, according to Guthnick (*A.N.* 4903). Fl. 20 is variable to the extent of 0·20 according to Danjon (*C.R.*, 171, 19).

In passing it may be mentioned here that Table I., in which the stars are arranged according to spectral type, shows a systematic difference between Harvard Photometry and Revised Harvard Photometry depending upon spectrum or colour. The positive residuals in column 4 correspond to the whiter B and A stars, whilst the redder K stars give negative residuals. What we spoke of before as corrections to H.P. are in most cases merely due to an adjustment to the different colour scale of H.R., the original measures made by the late Professor E. C. Pickering remaining practically unaltered.

It having appeared that the observations were sufficiently refined to reveal small differences between the Harvard catalogues, it was decided to use them to inquire into the nature and magnitude of the accidental errors affecting the method of observation.

For this purpose were taken all the observations made with the naked eye, and consisting of four or more comparisons, from 1920 Aug. 23 to Sept. 8 inclusive. There are 100 comparisons, forming 19 observations. No comparison was omitted, however discordant.

Although not very numerous, it was thought that this series of observations might yield good results, since some of the disturbing factors productive of errors and discrepancies in ordinary variable star observation would be absent or much reduced. (1) Personal differences, instrumental equation, and seasonal effect may be ruled out. (2) The magnitudes of the comparison stars have been ascertained with much greater accuracy than is the case with most variable star sequences. (3) The observer remained ignorant of the reputed magnitudes of comparison stars whilst the observations were being made. (4) There could be little bias due to any knowledge of magnitude or probable tendency of the *Nova* which it was known might exhibit rapid oscillations in brightness. (5) More than usual pains were taken to ensure accuracy.

An observation consists of a comparison with, on the average, 5 stars. From each comparison an independent value for the magnitude of the *Nova* is deduced. The residual differences found by subtracting the mean from the several separate values form the basis of the following discussion.

Besides Harvard Photometry, the observations were reduced, using magnitudes taken from Revised Harvard Phot., Potsdam (P.D.), and Uranometria Nova Oxoniensis. The magnitudes of the *Nova* were calculated, firstly, on the supposition that the grade or step was one-tenth of a magnitude, and secondly, using a corrected value for the grade. The mean of the residuals, or mean divergence, using the several catalogues, is given below :—

Grade.	H.P.	H.R.	P.D.	U.N.O.
Uncorrected	·096	·089	·108	·118
Corrected	·075	·072	·093	·106

The measure of agreement with the Revised Harvard Photometry is shown in Table II. The residuals are small, and there is no indication of systematic colour difference. Three of the larger residuals are supported by Potsdam; the remainder refer to stars observed on two nights, or stars difficult to observe.

(In Tables I. and II. a few observations made after Sept 8 (partly with binoculars) are used, but these are not included in the general discussion.) H.R. magnitudes are used in what follows, since they evidently suit the observations best.

For comparison, although the cases are not strictly parallel, reference may be made to the polarising photometer residuals given in the Harvard catalogues:—

The mean of the residuals on the first page of Harvard Photometric Durchmusterung	= 0·063
Mean of "Photometric Residuals" on first page of Revised Harvard Photometry	= 0·071
H.P. does not give residuals, but probable errors: mean of P.E.'s on first page of H.P.	= 0·074
Mean of above 100 Argelander residuals	= 0·072
Mean P.E. of 19 Argelander observations	= 0·027

It is obvious that a very large light interval cannot be estimated with the same degree of precision as a smaller one.

In order to see in what way the size of light interval affects the accuracy of the estimates, the residuals were arranged in groups according to estimated light interval and the means of these groups taken. When divided up in this way the number of residuals is too small to give very even results, but by taking overlapping means of adjacent groups, points were obtained through which a fairly smooth curve could be drawn. For this purpose it would not be safe to rely upon the residuals based upon corrected step values, since any over-correction might lead to a fictitious accordance. On the other hand, if the grade is known to be not exactly one-tenth, it would not be fair on the wider estimates to adopt the uncorrected values. Both were consequently used, and curves based upon the two sets of residuals are shown in fig. 1. A curve drawn about midway between them is probably a safe representation of the facts.

It might have been expected that the residuals would increase or decrease regularly with the light intervals, but the curve shows that this is not the case. The largest residuals do certainly occur with the wider estimates, and at first the curve falls as the intervals become smaller; but the minimum is reached at about 4 grades, the curve rising again towards zero, showing that the estimates of equality and of very small intervals were actually less accurate than those of larger ones such as 3, 4, and 5 grades.

In the case of the zero interval (estimate of equality) there is a personal scale effect, to which most observers are probably subject, due to the circumstance that when undecided an observer, through a conscious or unconscious tendency towards simplicity, will incline to

give the benefit of the doubt to the zero estimate, which will show, consequently, larger residuals. Still, it would be thought that the estimates of small intervals such as 1 grade would be more accurate than those of 4 or 5 grades, but the latter give somewhat smaller residuals, as shown.

It cannot be supposed that the difficulty of estimating the interval actually increases through a diminution in the size of the interval; so

Fig. 1.—Estimated Light Interval in Grades.

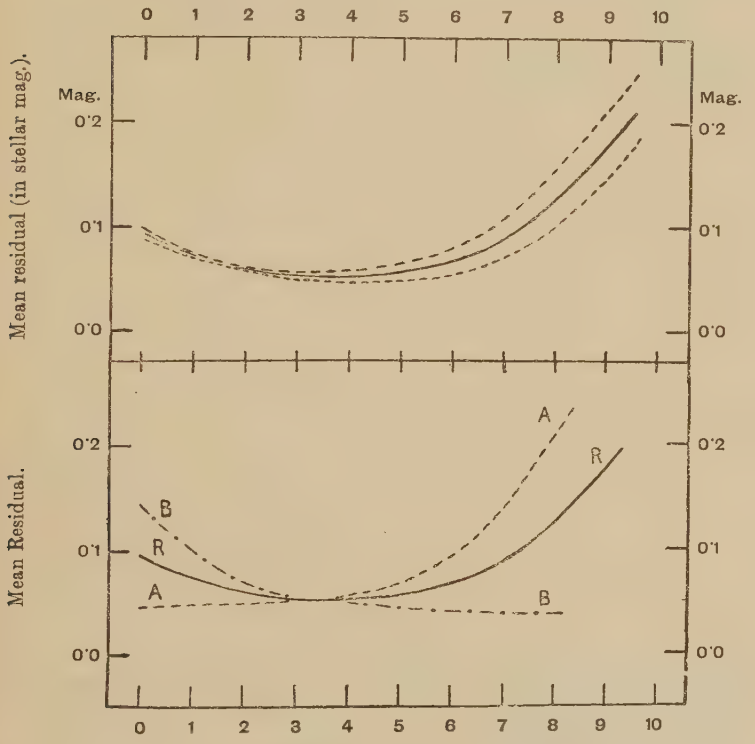


Fig. 2.—Estimated Light Interval in Grades.

it must be assumed that some other factor comes into play. The curve thus requires resolution into two (or more) components.

The material in hand is not sufficient to enable a definite conclusion to be arrived at, but a general view of the solution may be seen in fig. 2. The continuous line R is the residual curve (middle curve of fig. 1). Curve A represents the way in which the accidental errors increase on account of the increasing difficulty of estimating the larger intervals. Curve B represents the other factor, which accounts for the shape of the resultant curve R.

What is this other factor?

Ideal conditions for an observation might obtain if there were available for comparisons several stars differing little from the variable in brightness and situated at a conveniently small angular distance from it. Such ideal circumstances seldom occur, so the observer has to make concessions in the two directions of larger light intervals and greater separation, but he will naturally avoid both difficulties being present to a considerable degree in any one comparison. Consequently, the mean angular distance from the variable of the comparison stars used differing little from it in brightness will be greater than that of the stars differing much. The accidental errors due to angular separation will be greatest at zero, and will diminish with increasing light interval somewhat in the manner indicated by curve B.

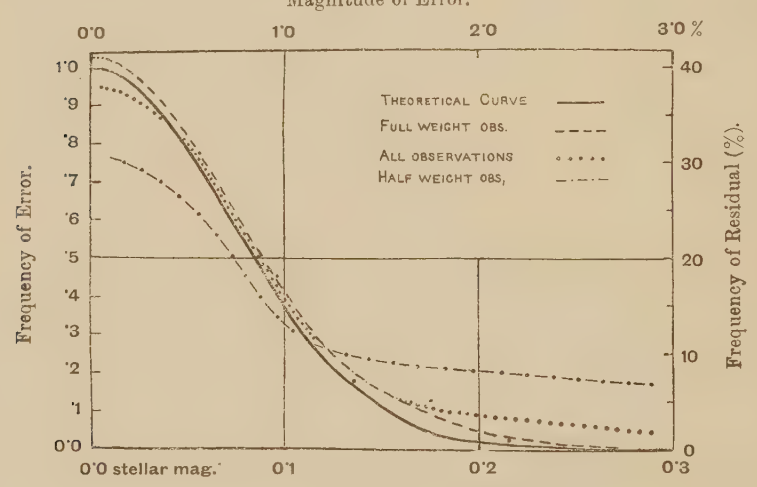


Fig. 3.—Size of Residual in Stellar Mag.

At the time that the observations were made class numbers (not exactly weights) were affixed to them as a guide to the order of relative reliability, and the class number once finally assigned to a comparison has in no case been altered.

The results referred to up to the present were obtained from a consideration of *all* the comparisons, some of which were made under unfavourable atmospheric conditions or for other reasons were believed to be inferior. When the residuals are arranged according to their class numbers they are found fully to support them. The mean of the residuals from the best class (5) of comparisons is 0·043, from class (4) 0·061, class (3) 0·067, and of the worst class (2) 0·120. Thus, though there are individual exceptions, these figures show that on the whole the observer was correct in his estimates of the order, as regards accuracy, in which they should be placed. The comparisons of the three better classes may conveniently be grouped together as good or full-weight comparison, those of the remaining class (2) being considered as inferior

or half weight. The mean of the residuals of the first group is 0·06, of the second 0·12.

The residuals have also been arranged in groups according to size, the first group containing those of 0·00 to 0·03, the second those of 0·04 to 0·07, and so on. The number of residuals in each group ($\times \frac{8}{7}$ in the case of the first group, which is smaller than the others), expressed in per cent., was plotted against the group size and a smooth curve drawn through the points. Separate curves are drawn for full-weight, half-weight, and all comparisons, and these curves are compared with the

TABLE III.

Size of Residual. Mag.	Original Class of Comparison. 5· No.	5· %	4· No.	4· %	3· No.	3· %	2· No.	2· %	All. No.	All. %	5–4–3· No.	5–4–3· %
0·00–0·03	4	(65)	20	(43)	5	(24)	4	(26)	33	(40)	29	(40)
·04– ·07	1	14	15	30	12	52	5	30	33		28	34
·08– ·11	1	14	10	19	5	21	1	6	17		16	20
·12– ·15	1	14	3	10	0	0	2	12	6		4	5
·16– ·19	...		3	10	1	3	0	0	4		4	5
·20– ·23	...		1	2	0	0	2	12	3		1	1
·24– ·27	...		...		1	3	2	12	3		1	1
·28– ·31	...		...		...		0	0	0			
·32– ·35	...		...		...		0	0	0			
0·36–0·39	...		...		...		1	6	1			
All	7		52		24		17		100		83	

TABLE IV.

0·00–0·10	6	85	40	77	21	92	9	53	77		68	82
>0·10	1	15	12	23	2	8	8	47	23		15	18
>0·20	0	0	0	0	1	4	5	30	5		1	1

theoretical frequency curve based on the law of probabilities of Laplace. The scale of the frequency curve of the full-weight observations is adjusted so as to be comparable with the theoretical curve, which it resembles closely; the others, which are also slightly out of scale, depart from it principally in the lower portion, showing that the larger errors are rather more frequent than theory would suggest, as is always found to be the case when actual observations are compared with the theoretical distribution.

From the frequency curves we can find the probability of errors of various magnitudes occurring. Thus with full-weight comparisons the chances of the error falling in a group centred at 0·1 mag. are 100 to 16 against, at 0·2 mag. 50 to 1 against.

The grade of about one-tenth of a magnitude being much employed by observers as a unit, the number of residuals exceeding and not exceeding that quantity are shown in Table IV. Full-weight comparisons

give 82 per cent. not greater than one unit, and 1 per cent. greater than two units.

The shape of the lower portion of these curves indicates the presence of some errors other than accidental; these may be systematic errors, mistakes of identity or of entry, etc.

We may now form an idea of the magnitude of the accidental errors affecting observations such as those examined.

The comparisons of the three better classes, which constitute 83 per cent. of the whole, may be taken as the standard of normal full-weight comparisons. The mean of the residuals of these is 0·06, but we shall have amongst these: (1) residuals due to errors in the catalogue magnitudes of the comparison stars; (2) residuals due to want of perfect adjustment of colour scale; (3) probably also a few large residuals as indicated by the shape of the frequency curve, not due to accidental errors. Thus the purely accidental errors can hardly be responsible for a mean residual greater than 0·05.

The probable error of an observation consisting of five such estimates would be in the neighbourhood of 0·02 mag., which is insignificant in comparison with the much larger constant and personal errors which affect all determinations of visual star magnitude, whether made by means of a polarising photometer, a wedge photometer, or by the differential method of Argelander.

Observations of Minor Planets made with the 10-inch Refractor of the University Observatory, Utrecht, Holland. By J. van der Bilt.

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
			(1) *Ceres*, 1920–1921.				
Sept. 28	15 30 17	6 22 3·47	9·351$_n$	+21 53 25·6	0·681	4	2 1
29	13 35 14	6 22 54·87	9·540$_n$	+21 54 57·8	0·743	10	5 2
Oct. 4	12 22 7	6 27 17·13	9·580$_n$	+22 3 31·5	0·779	6	3 3
5	12 51 29	6 27 58·72	9·562$_n$	+22 5 21·0	0·758	6	3 3
7	14 34 20	6 29 44·91	9·418$_n$	+22 9 10·5	0·692	8	4 4
9	13 9 44	6 31 13·17	9·536$_n$	+22 12 2·5	0·737	6	3 5
13	13 37 12	6 34 0·64	9·486$_n$	+22 20 56·2	0·711	10	5 5
18	14 12 25	6 37 0·46	9·388$_n$	+22 31 55·4	0·681	12	6 6
22	13 28 28	6 38 57·77	9·451$_n$	+22 41 28·1	0·696	10	5 7
23	12 53 21	6 39 22·97	9·501$_n$	+22 43 57·9	0·715	12	3 7
24	12 33 12	6 39 47·37	9·522$_n$	+22 46 29·3	0·724	8	4 8
26	13 20 38	6 40 32·27	9·441$_n$	+22 51 57·6	0·691	10	5 9
28	13 37 40	6 41 3·51	9·389$_n$	+23 1 27·0	0·675	6	3 9
29	11 36 37	6 41 18·68	9·560$_n$	+23 3 5·4	0·747	6	3 9

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.		Comp. Star.
	h m s	h m s		° ′ ″				
Nov. 1	13 15 29	6 42 7·84	9·407$_n$	+23 9 32·1	0·678	10	5	10
7	12 32 32	6 42 42·77	9·448$_n$	+23 9 26·9	0·686	10	5	11
17	14 58 50	6 41 18·15	7·923	+24 8 45·2	0·615	6	3	12
19	13 37 27	6 40 57·52	9·065$_n$	+24 7 51·2	0·625	8	3	12
22	12 38 50	6 39 31·11	9·276$_n$	+24 30 6·5	0·637	12	6	13
Jan. 14	14 56 9	5 52 29 86	9·584	+28 18 3·0	0·718	6	3	14
16	13 9 34	5 50 58·80	9·466	+28 23 13·7	0·634	16	4	15
22	10 54 27	5 46 56·16	9·117	+28 37 17·1	0·561	16	4	16
Feb. 16	13 17 27	5 41 46·01	9·609	+29 17 31·2	0·749	10	5	17
19	10 52 36	5 42 28·32	9·479	+29 20 53·5	0·626	12	3	17
23	11 43 7	5 43 52·65	9·566	+29 25 16·1	0·684	6	3	17
26	11 39 55	5 45 14·53	9·573	+29 28 14·5	0·691	2	1	18
Mar. 12	11 25 31	5 54 44·82	9·597	+29 39 16·8	0·718	10	5	19
20	12 50 28	6 2 22·26	9·610	+29 43 9·3	0 812	6	3	20
Apr. 9	9 51 2	6 26 28·20	9·586	+29 41 47·6	0·704	8	4	21
16	10 9 55	6 36 26·47	9·607	+29 36 27·9	0·738	12	6	22
28	9 37 22	6 54 49·29	9·605	+29 20 24·9	0·737	10	5	23
30	9 55 59	6 58 3·06	9·612	+29 15 48·3	0·757	10	5	23

(2) *Pallas*, 1920.

July 18	13 34 29	1 53 28·94	9·524$_n$	+ 1 27 30·5	0·837	6	3	24
19	13 38 39	1 54 23·32	9·518$_n$	+ 1 22 16·4	0·837	6	3	24
Sept. 16	14 17 4	2 19 35·44	8·497$_n$	− 9 59 43·8	0·890	10	5	25
20	14 2 8	2 18 36·74	8·457$_n$	−11 9 51·8	0·894	10	5	26
23	13 0 18	2 17 39·08	9·027$_n$	−12 2 38·3	0·895	10	5	27
28	14 46 12	2 15 32 29	8·987	−13 33 54·9	0·901	6	3	28
Oct. 4	11 54 11	2 12 23·67	9·130$_n$	−15 18 56·6	0·894	10	5	29
5	11 57 56	2 11 40·67	9·087$_n$	−15 36 36·6	0·906	12	4	30
6	12 37 27	2 11 8·87	8·721$_n$	−15 54 39·6	0·910	6	3	30
8	13 17 12	2 9 50·31	8·437	−16 29 48·0	0·913	6	3	31
9	12 40 51	2 9 11·06	8·393$_n$	−16 46 29·6	0·914	12	6	32
10	13 6 51	2 8 28·92	8·409	−17 3 46·9	0·915	10	5	33
12	14 18 50	2 7 1·44	9·191	−17 37 55·6	0·909	10	5	34
13	13 9 39	2 6 20·06	8·727	−17 53 30·2	0 916	10	5	35
18	12 52 41	2 2 31·46	8·808	−19 11 45·1	0·919	10	5	36
21	13 14 59	2 0 8·26	9·093	−19 55 47·9	0·917	12	5	37
22	12 51 43	1 59 21·05	8·973	−20 9 36·0	0·920	6	3	38
23	12 29 44	1 58 33·91	8·819	−20 23 2·8	0·922	10	5	39
24	12 15 42	1 57 46·23	8·705	−20 36 15·5	0·923	10	5	40

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
Oct. 26	13 2 45	1 56 6·39	9·155	−21 2 41·7	0·917	12	6 41
28	10 41 2	1 54 34·32	8·884$_n$	−21 25 56·9	0·924	12	6 42
29	11 15 21	1 53 44·77	8·043$_n$	−21 37 55·7	0·926	10	5 42
Nov. 1	10 39 21	1 51 22·76	8·682$_n$	−22 10 31·6	0·927	10	5 43
4	11 8 22	1 49 1·94	8·401	−22 40 55·1	0·929	12	6 44
8	10 56 13	1 46 4·16	8·561	−23 13 35·8	0·929	12	6 45
16	11 34 30	1 40 48·44	9·207	−24 3 42·0	0·921	4	2 46
21	10 3 35	1 38 11·81	8·678	−24 21 51·3	0·931	6	3 47
21	10 13 36	1 38 11·63	8·812	−24 21 50·4	0·930	6	3 48
23	9 31 34	1 37 17·45	7·880	−24 26 40·1	0·932	6	3 49
23	9 44 40	1 37 17·44	8·480	−24 26 39·0	0·931	6	3 47
		(4) *Vesta*, 1920.					
July 15	11 0 53	17 21 3·45	9·087	−21 20 4·1	0·921	10	5 50
16	10 35 39	17 20 33·43	8·947	−21 24 13·0	0·923	10	5 50
17	10 59 5	17 20 4·44	9·125	−21 28 27·7	0·920	6	3 50
24	11 24 44	17 17 39·10	9·338	−21 57 37·0	0·908	6	3 51
Aug. 16	9 6 9	17 21 39·86	9·149	−23 26 36·8	0·922	12	6 52
Oct. 20	6 4 21	18 45 6·87	9·107	−25 37 55·4	0·927	8	4 53
29	5 49 16	19 1 34·99	9·127	−25 28 14·5	0·926	8	4 54
31	5 54 17	19 5 21·20	9·168	−25 24 53·0	0·925	12	6 55
		(5) *Astraea*, 1921.					
Feb. 26	11 13 40	11 35 23·54	9·247$_n$	+ 6 59 45·5	0·801	8	4 56
Mar. 10	13 18 57	11 26 17·03	9·010	+ 8 43 39·6	0·783	6	3 57
14	13 23 19	11 23 9·35	9·138	+ 9 16 11·9	0·781	8	4 58
Apr. 18	12 11 20	11 7 5·82	9·396	+11 47 16·8	0·777	6	3 59
		(6) *Hebe*, 1921.					
Feb. 21	12 47 38	7 52 7·77	9·427	+17 25 16·4	0·738	6	3 60
23	13 1 12	7 51 14·90	9·466	+17 40 42·4	0·746	6	3 61
24	9 57 4	7 50 54·33	8·578	+17 47 18·7	0·695	8	4 62
		(9) *Metis*, 1921.					
Feb. 21	13 23 3	8 45 46·80	9·421	+27 16 20·6	0·632	6	3 63
24	8 18 43	8 43 46·93	9·333$_n$	+27 19 45·6	0·608	6	3 63
Mar. 12	12 57 28	8 37 7·22	9·521	+27 9 51·6	0·678	6	3 64
Apr. 30	12 43 9	9 5 24·33	9·592	+23 15 5·7	0·811	4	2 65

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
		(10) *Hygeia*, 1921.					
Feb. 24	10 37 12	9 55 34·10	8·984$_n$	+ 8 23 51·9	0·785	6 4	66
Mar. 10	13 57 10	9 45 14·42	9·454	+ 9 12 53·5	0·799	8 4	67
14	12 59 6	9 42 49·61	9·380	+ 9 25 8·2	0·790	6 3	68
		(11) *Parthenope*, 1920.					
Sept. 29	12 56 44	0 39 45·97	8·899	− 3 39 53·7	0·861	8 3	69
Oct. 4	9 19 41	0 35 34·63	9·321$_n$	− 4 11 16·1	0·859	6 3	70
5	9 59 35	0 34 41·61	9·175$_n$	− 4 17 33·6	0·862	6 3	70
6	12 13 36	0 33 45·17	8·809	− 4 24 5·2	0·864	6 3	70
8	14 26 33	0 31 59·43	9·414	− 4 36 0·6	0·858	6 3	71
11	10 49 37	0 29 31·03	8·478$_n$	− 4 51 11·2	0·867	10 5	72
14	12 32 0	0 27 14·76	9·174	− 4 59 20·3	0·866	10 5	73
22	12 15 2	0 21 46·40	9·249	− 5 34 49·5	0·867	10 5	74
23	12 5 5	0 21 11·18	9·230	− 5 37 25·5	0·867	6 3	75
24	11 53 56	0 20 37·46	9·206	− 5 39 48·8	0·868	6 3	75
26	11 29 47	0 19 33·66	9·142	− 5 43 57·5	0·869	6 3	75
28	10 22 46	0 18 37·46	8·701	− 5 47 8·5	0·871	10 5	75
29	10 11 34	0 18 10·54	8·596	− 5 48 23·8	0·872	10 5	75
31	9 25 13	0 17 22·51	8·303$_n$	− 5 50 16·7	0·872	6 3	76
Nov. 17	12 5 49	0 14 56·43	9·468	− 5 30 46·6	0·858	10 4	77
		(13) *Egeria*, 1920.					
Oct. 12	11 47 58	0 16 47·36	8·956	−15 34 1·8	0·908	6 3	78
18	11 14 43	0 11 20·59	8·923	−15 19 43·7	0·908	6 3	79
19	11 22 44	0 10 29·49	9·017	−15 16 22·2	0·907	12 3	79
		(14) *Irene*, 1921.					
Feb. 24	11 19 29	6 53 9·21	9·437	+30 36 2·0	0·590	6 3	80
26	10 32 21	6 53 15·21	9·332	+30 36 56·9	0·557	6 3	80
Mar. 14	12 30 25	6 59 14·70	9·606	+30 31 30·0	0·721	6 3	81
		(16) *Psyche*, 1920–1921.					
Nov. 7	11 28 32	4 50 10·25	9·320$_n$	+17 43 55·6	0·719	6 3	82
9	13 7 23	4 49 3·92	8·632$_n$	+17 40 5·9	0·697	6 3	83
17	14 34 19	4 42 51·43	9·195	+17 23 34·9	0·715	6 3	84
21	10 58 41	4 39 33·37	9·195$_n$	+17 15 48·8	0·713	6 3	85
Jan. 16	9 41 38	4 5 4·27	9·107	+16 49 9·8	0·713	12 3	86
Feb. 24	8 49 2	4 23 45·77	9·392	+18 36 42·1	0·721	6 3	87

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
		(20) *Massalia*, 1921.					
Feb. 19	12 7 9	8 14 37·77	9·243	+18 39 7·3	0·703	12	3 88
24	10 58 41	8 12 14·40	9·017	+18 49 13·6	0·689	6	3 89
Mar. 14	12 10 49	8 10 46·87	9·478	+19 3 54·6	0·738	6	3 90
		(21) *Lutetia*, 1920.					
Sept. 20	10 51 54	21 40 25·52	9·057	−19 35 49·4	0·918	6	3 91
Oct. 7	9 39 57	21 42 6·78	9·011	−19 2 21·7	0·917	6	3 92
31	8 54 27	21 56 45·01	9·189	−16 55 11·8	0·907	6	3 93
		(23) *Thalia*, 1920.					
Nov. 3	13 9 8	2 13 13·83	9·213	+ 4 2 18·0	0·818	6	3 94
7	10 34 32	2 9 17·25	8·636$_n$	+ 3 59 13·3	0·816	6	3 95
		(25) *Phocaea*, 1920.					
July 16	11 10 2	15 4 54·74	9·475	+ 5 1 22·2	0·822	4	2 96
		(30) *Urania*, 1920–1921.					
Oct. 19	10 14 12	3 0 5·25	9·423$_n$	+21 7 37·5	0·703	10	5 97
21	11 58 4	2 58 24 58	8·999$_n$	+21 1 54·7	0·662	6	3 98
22	13 55 40	2 57 29·45	9·019	+20 59 22·9	0·664	6	3 99
25	9 35 11	2 55 0·12	9·442$_n$	+20 50 38·7	0·712	10	5 100
26	11 5 32	2 54 1·33	9·161$_n$	+20 47 2·4	0·673	12	4 100
29	10 45 0	2 51 11·39	9·187$_n$	+20 36 4·8	0·676	10	5 101
31	9 44 0	2 49 16·31	9·358$_n$	+20 28 32·6	0·694	6	3 101
Nov. 1	13 44 24	2 48 6·34	9·211	+20 23 19·9	0·681	6	3 102
17	14 13 20	2 32 35 49	9·477	+19 5 59·6	0·738	12	3 103
22	13 5 40	2 28 33·75	9·397	+18 41 37·6	0·721	10	5 104
Jan. 16	10 9 25	2 36 13·10	9·455	+17 49 38·4	0·741	6	3 105
		(32) *Pomona*, 1920.					
Oct. 5	8 40 29	22 33 29·50	8·941$_n$	− 2 22 15·3	0·854	4	2 106
6	11 4 25	22 32 35·82	9·152	− 2 26 46·1	0·854	6	3 107
		(34) *Circe*, 1921.					
Apr. 4	13 52 15	12 59 12 03	9·200	− 4 30 35·9	0·863	8	3 108
11	13 25 14	12 53 46·37	9·222	− 3 38 17·8	0·859	6	3 109
15	11 48 40	12 50 50·98	8·701	− 3 10 10·4	0·858	12	3 110

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.		Comp. Star.
	h m s	h m s		° ′ ″				
		(39) *Laetitia*, 1921.						
Apr. 2	11 59 16	13 45 51·16	8·993	+ 0 19 55·5	0·843	8	4	111
4	10 14 23	13 44 29·11	9·363$_n$	+ 0 33 53·4	0·836	6	3	112
15	10 42 8	13 36 10·76	9·090$_n$	+ 1 50 18·5	0·830	10	5	113
29	9 47 52	13 25 57·91	9·029$_n$	+ 3 9 42·8	0·822	6	3	114
		(43) *Ariadne*, 1920.						
Nov. 3	12 50 15	2 1 42·64	9·056	+17 7 14·1	0·708	8	4	115
4	9 48 5	2 0 48·24	9·095	+17 0 53·2	0·711	8	4	116
		(48) *Doris*, 1921.						
Apr. 4	13 30 10	13 43 29·68	8·776	− 7 30 40·2	0·879	12	3	117
11	13 44 28	13 38 35·80	9·117	− 6 24 30·7	0·872	6	3	118
15	11 8 51	13 35 48·91	8·918$_n$	− 6 26 41·2	0·874	6	3	119
		(56) *Melete*, 1920.						
Oct. 8	14 49 47	1 10 16·96	9·387	+ 6 32 42·7	0·808	8	4	120
		(60) *Echo*, 1921.						
Feb. 24	9 12 0	8 17 43·68	8·897$_n$	+14 19 41·7	0·733	8	4	121
Mar. 12	10 56 8	8 16 33·99	9·270	+15 18 12·0	0·737	6	3	122
		(64) *Angelina*, 1920.						
Oct. 8	12 37 51	23 14 5·80	9·349	− 3 23 28·3	0·856	6	3	123
12	11 8 42	23 11 47·47	9·100	− 3 38 15·1	0·860	6	3	124
Nov. 8	10 11 46	23 5 49·79	9·308	− 4 24 27·9	0·860	12	4	125
		(72) *Feronia*, 1920.						
Oct. 10	13 43 17	0 15 17·18	9·378	+ 4 51 42·8	0·817	12	4	126
11	11 17 42	0 14 36·69	8·576	+ 4 43 33·3	0·811	10	5	126
		(80) *Sappho*, 1920.						
Oct. 7	9 17 48	19 42 36·38	9·371	−10 14 17·0	0·878	12	3	127
12	9 57 19	19 49 45·62	9·466	−10 25 40·0	0·870	8	4	128
		(91) *Ægina*, 1920.						
Oct. 18	12 32 14	0 13 40·65	9·278	+ 1 33 31·3	0·833	10	5	129
19	12 49 38	0 12 56·66	9·339	+ 1 29 48·1	0·833	10	5	129
Nov. 8	11 22 33	0 3 18·42	9·342	+ 0 45 25·1	0·837	12	3	130

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
		(95) *Arethusa*, 1920.					
Sept. 20	12 21 8	21 32 59·50	9·380	+ 5 42 44·1	0·812	6	3 131
		(107) *Camilla*, 1920.					
Nov. 2	10 18 51	1 48 41·45	8·821$_n$	+ 2 46 23·7	0·824	6	3 132
3	12 51 1	1 47 59·03	9·237	+ 2 40 57·0	0·826	4	2 133
		(114) *Kassandra*, 1920.					
Nov. 7	11 51 17	1 32 39·25	9·121	+ 3 58 40·7	0·818	6	3 134
		(115) *Thyra*, 1921.					
Apr. 2	10 16 40	11 24 58·51	8·612$_n$	− 12 12 34·8	0·898	6	3 135
4	9 50 34	11 23 17·90	8·828$_n$	− 12 1 54·9	0·897	12	4 135
		(144) *Vibilia*, 1921.					
Mar. 15	12 41 31	9 49 5·29	9·354	+ 20 6 7·9	0·700	12	3 136
		(200) *Dynamene*, 1920.					
Oct. 5	10 25 2	22 42 31·90	8·791	− 2 41 30·0	0·856	6	3 137
6	11 22 12	22 41 58·45	9·188	− 2 43 19·0	0·854	6	3 138
		(216) *Cleopatra*, 1921.					
Feb. 23	12 26 18	7 27 13·83	9·429	+ 3 1 28·6	0·828	8	3 139
24	10 14 24	7 27 1·80	8·999	+ 3 7 10·8	0·823	12	5 139
		(234) *Barbara*, 1920.					
Oct. 7	10 18 11	0 24 31·05	9·009$_n$	− 20 58 34·2	0·921	6	3 140
8	11 44 50	0 23 56·81	8·702	− 21 7 22·1	0·925	10	5 141
12	10 39 14	0 22 0·70	8·484$_n$	− 21 34 58·2	0·925	6	3 142
		(308) *Polyxo*, 1920.					
Oct. 20	12 40 18	0 7 54·62	9·338	− 0 11 17·9	0·842	6	3 143
		(385) *Ilmatar*, 1920.					
Oct. 8	12 49 44	23 24 16·16	9·352	+ 1 11 18·9	0·835	6	3 144
9	11 46 41	23 23 35·03	9·166	+ 1 9 23·1	0·836	6	3 144
		(451) *Patientia*, 1920.					
Nov. 17	11 41 2	1 51 10·41	9·172	− 8 4 19·3	0·879	6	3 145
19	13 14 4	1 49 54·38	9·443	− 7 57 6·0	0·866	6	3 146
22	11 45 9	1 48 14·67	9·270	− 7 45 25·4	0·875	6	3 147

Date.	Utrecht M.T.	App. R.A.	Log Parallax Factor.	App. Decl.	Log Parallax Factor.	No. of Obs.	Comp. Star.
	h m s	h m s		° ′ ″			
		(532) *Herculina*, 1920.					
Nov. 9	12 36 2	2 25 8·61	9·128	− 9 5 57·7	0·883	12 3	148
19	11 59 43	2 17 12·99	9·176	− 9 3 52·2	0·883	12 5	149
21	11 25 53	2 15 46·98	9·054	− 9 1 1·0	0·884	6 3	149
		(690) *Wratislavia*, 1920.					
Oct. 10	14 54 53	0 22 1·06	9·509	+19 34 36·6	0·745	12 4	150
11	11 42 31	0 21 41·76	8·481$_n$	+19 27 51·3	0·676	8 4	151

Observations of Brightness (Durchm. Scale).

(1) *Ceres*, 1920 Sept. 28, $8^m·9$; Sept. 29, $8^m·9$; Oct. 5, $8^m·3$; Oct. 7, $9^m·0$; Oct. 13, $8^m·6$; Oct. 18, $8^m·7$; Oct. 22, $8^m·6$; Oct. 23, $8^m·9$; Oct. 26, $8^m·9$; Oct. 28, $8^m·9$; Nov. 1, $8^m·4$; Nov. 7, $8^m·2$; Nov. 19, $8^m·6$; Nov. 22, $8^m·1$. 1921 Jan. 14, $7^m·8$; Jan. 16, $7^m·9$; Feb. 16, $7^m·9$; Feb. 19, $8^m·1$; March 12, $7^m·8$; April 16, $8^m·7$; April 28, $8^m·9$; April 30, $8^m·9$.
(2) *Pallas*, 1920 Sept. 16, $8^m·3$; Oct. 8, $7^m·9$; Oct. 10, $8^m·0$; Oct. 18, $7^m·9$; Oct. 22, $8^m·1$; Oct. 28, $7^m·6$; Nov. 1, $7^m·9$; Nov. 8, $8^m·2$; Nov. 21, $8^m·4$.
(4) *Vesta*, 1920 July 15, $6^m·6$; July 16, $6^m·5$; Aug. 16, $7^m·0$; Oct. 29, $7^m·7$.
(5) *Astraea*, 1921 March 10, $9^m·0$; March 14, $8^m·6$.
(10) *Hygeia*, 1921 March 10, $9^m·2$; March 14, $9^m·2$.
(11) *Parthenope*, 1920 Oct. 4, $9^m·0$; Oct. 5, $9^m·0$; Oct. 6, $9^m·1$; Oct. 11, $9^m·0$; Oct. 22, $9^m·2$.
(14) *Irene*, 1921 Feb. 24, $9^m·4$; Feb. 26, $9^m·4$.
(16) *Psyche*, 1920 Nov. 9, $9^m·2$; Nov. 21, $9^m·1$.
(20) *Massalia*, 1921 Feb. 19, $8^m·8$; Feb. 24, $8^m·9$.
(30) *Urania*, 1920 Oct. 19, $9^m·3$; Oct. 25, $9^m·1$; Oct. 29, $9^m·1$; Nov. 1, $9^m·1$; Nov. 22, $9^m·0$.
(39) *Laetitia*, 1921 April 2, $9^m·4$; April 29, $9^m·1$.
(56) *Melete*, 1920 Oct. 8 and 9. Very faint, not brighter than 12^m.
(144) *Vibilia*, 1921 March 15. Very faint, not brighter than 12^m.
(532) *Herculina*, 1920 Nov. 21, $9^m·7$.

Comparison Stars.

No.	Mean R.A.	Red. to Date.	Mean Decl.	Red. to Date.	Reference.
	h m s	s	° ′ ″	″	
1	6 20 54·68	+3·42	+21 41 25·6	− 7·8	A. G. Berl. B. 2373
2	6 23 9·03	+3·44	+21 52 8·5	− 8·1	,, ,, 2401
3	6 27 10·30	+3·60	+22 14 35·4	− 8·7	,, ,, 2433
4	6 29 18·21	+3·68	+22 11 11·9	− 9·0	,, ,, 2455
5	6 32 40·39	+3·71	+22 13 2·2	− 9·3	,, ,, 2480
6	6 37 51·46	+3·97	+22 34 12·5	−10·4	,, ,, 2538
7	6 39 35·09	+4·08	+22 39 38·7	−10·7	,, ,, 2557
8	6 40 13·69	+4·14	+22 52 21·0	−11·0	,, ,, 2568
9	6 40 6·38	+4·20	+22 55 10·8	−11·0	,, ,, 2564
10	6 43 26·94	+4·39	+23 18 43·1	−11·9	,, ,, 2601

Comparison Stars—continued.

No.	Mean R.A.	Red. to Date.	Mean Decl.	Red. to Date.	Reference.
	h m s	s	° ′ ″	″	
11	6 41 19·21	+4·58	+23 27 15·5	−12·0	A. G. Berl. B. 2578
12	6 40 44·38	+4·91	+24 12 40·2	−12·7	,, ,, 2575
13	6 39 14·21	+5·05	+24 37 47·5	−12·7	,, ,, 2553
14	5 55 5·78	+2·33	+28 7 16·9	− 7·2	,, Cambr. 2843
15	5 49 58·04	+2·33	+28 22 25·1	− 6·8	,, ,, 2776
16	5 47 21·07	+2·31	+28 36 48·4	− 6·4	,, ,, 2744
17	5 42 7·84	+2·03	+29 24 4·0	− 5·0	,, ,, 2688
18	5 47 13·58	+1·88	+29 29 54·6	− 5·0	,, ,, 2742
19	5 53 21·50	+1·63	+29 37 9·1	− 5·1	,, ,, 2822
20	6 1 19·98	+1·50	+29 31 11·0	− 5·4	,, ,, 2943
21	6 24 21·84	+1·20	+29 31 55·7	− 6·3	,, ,, 3268
22	6 37 29·30	+1·13	+29 30 28·2	− 6·6	,, ,, 3441
23	6 56 12·66	+1·01	+29 19 41·7	− 7·2	,, ,, 3670
24	1 51 45·77	+1·94	+ 1 27 16·9	+13·9	Boss 435
25	2 21 18·08	+3·61	− 9 57 43·1	+21·5	A. G. Cbr. Mass. 542
26	2 18 6·33	+3·69	−11 8 24·7	+22·0	,, ,, ,, 531
27	2 16 50·43	+3·74	−11 58 24·0	+22·0	,, ,, ,, 525
28	2 20 37·49	+3·77	−13 39 19·7	+21·7	,, ,, ,, 538
29	2 9 28·78	+3·89	−15 16 54·1	+22·0	A. G. Wash. 575
30	2 11 32·96	+3·91	−15 41 46·4	+22·8	,, ,, 583
31	2 10 8·89	+3·94	−16 21 41·2	+22·4	,, ,, 577
32	2 8 42·49	+3·94	−16 45 56·4	+21·7	,, ,, 574
33	2 10 17·03	+3·93	−17 15 50·1	+21·5	,, ,, 579
34	2 6 27·48	+3·95	−17 34 26·0	+21·3	,, ,, 565
35	2 4 56·84	+3·96	−17 46 12·5	+21·3	,, ,, 557
36	2 4 11·69	+4·00	−19 17 34·2	+20·6	Arg. W. 1093
37	2 0 31·51	+4·03	−19 45 1·6	+20·4	,, 1053
38	2 5 31·01	+4·02	−19 56 50·0	+20·0	,, 1110
39	1 59 59·10	+4·03	−20 16 6·0	+20·0	Cape Durchm. 234 *
40	1 55 57·80	+4·03	−20 35 36·0	+20·1	,, ,, 228 *
41	1 56 0·38	+4·02	21 12 46·0	+19·2	Arg. Gen. Cat. 1962, Bo. VI., −21° 356
42	1 55 8·48	+4·03	−21 31 24·8	+19·2	Bo. VI., −21° 352
43	1 52 11·46	+4·04	−22 12 58·3	+18·7	A. G. Cord. 891
44	1 47 52·98	+4·05	−22 33 48·6	+18·3	,, ,, 863
45	1 46 36·22	+4·03	−23 9 49·9	+17·3	,, ,, 856
46	1 44 58·07	+4·02	−24 16 5·0	+15·7	,, ,, 842

* Approximate position.

Comparison Stars—continued.

No.	Mean R.A.	Red. to Date.	Mean Decl.	Red. to Date.	Reference.
	h m s	s	° ′ ″	″	
47	1 38 32·83	+3·97	−24 17 16·7	+15·0	A. G. Cord. 797
48	1 36 33·32	+3·96	−24 15 3·8	+15·1	,, ,, 786
49	1 34 3·88	+3·94	−24 26 11·1	+14·7	,, ,, 770
50	17 19 54·79	+4·11	−21 22 5·9	+ 3·5	Boss 4413
51	17 20 20·63	+4·07	−21 49 14·7	+ 3·5	Arg. W. 13337
52	17 19 11·01	+3·90	−23 43 46·7	+ 2·8	A. G. Cord. 12005
53	18 44 43·58	+3·18	−25 42 15·8	+ 9·4	,, ,, 13067
54	18 59 51·19	+3·08	−25 21 0·9	+10·6	,, ,, 13251
55	19 3 52·81	+3·19	−25 12 20·4	+10·7	,, ,, 13301
56	11 34 28·81	+2·44	+ 6 56 16·5	−16·0	,, Lpz.$_2$ 5868
57	11 23 14·52	+2·35	+ 8 38 11·1	−16·6	,, ,, 5813
58	11 22 12·69	+2·36	+ 9 5 40·2	−16·6	,, ,, 5810
59	11 6 29·37	+2·21	+11 43 51·0	−14·7	,, Lpz.$_1$ 4216
60	7 50 29·87	+2·22	+17 26 16·2	−13·8	,, Berl. A. 3097
61	7 53 1·30	+2·21	+17 40 36·4	−13·8	,, ,, 3121
62	7 50 10·20	+2·20	+17 55 41·2	−13·6	,, ,, 3091
63	8 47 51·31	+2·41	+27 18 46·8	−13·9	,, Cbr. 4721
64	8 36 11·99	+2·24	+27 10 17·4	−12·0	,, ,, 4647
65	9 3 53·64	+1·52	+23 14 9·9	−10·4	,, Berl. B. 3665
66	9 55 46·21	+2·32	+ 8 18 52·8	−17·0	,, Lpz.$_2$ 5323
67	9 46 44·75	+2·30	+ 9 19 33·0	−17·1	,, ,, 5271
68	9 43 35·90	+2·23	+ 9 23 23·6	−17·1	,, ,, 5252
69	0 39 45·29	+4·13	− 3 31 1·9	+25·7	,, Strb. 153
70	0 35 4·74	+4·16	− 4 25 56·3	+25·9	,, ,, 131
71	0 29 20·11	+4·18	− 4 29 0·3	+26·0	,, ,, 114
72	0 28 15·79	+4·17	− 4 54 52·9	+25·3	,, ,, 112
73	0 30 24·53	+4·17	− 4 59 15·7	+25·6	,, ,, 117
74	0 23 0·54	+4·16	− 5 26 45·2	+25·4	,, ,, 95
75	0 18 9·38	+4·14	− 5 38 6·7	+25·4	A. G. Strb. 71; A. G. Ott. 62
76	0 15 46·62	+4·10	− 5 53 39·7	+24·9	,, Ott. 55
77	0 15 7·61	+3·98	− 5 29 34·3	+24·1	,, Strb. 60
78	0 15 18·80	+4·09	−15 16 35·8	+24·9	,, Wash. 79
79	0 10 35·23	+4·09	−15 14 55·8	+24·3	,, ,, 52
80	6 52 34·37	+2·20	+30 36 27·6	− 7·8	,, Lei. 2897
81	6 59 56·88	+1·91	+30 28 51·4	− 7·3	,, ,, 2958
82	4 51 33·78	+4·88	+17 52 44·7	− 8·0	A. G. Berl. A. 1352
83	4 47 12·82	+4·92	+17 33 23·9	+ 2·3	,, ,, 1330
84	4 45 41·37	+5·08	+17 23 4·2	+ 2·4	,, ,, 1322

Comparison Stars—continued.

No.	Mean R.A.	Red. to Date.	Mean Decl.	Red. to Date.	Reference.
	h m s	s	° ′ ″	″	
85	4 38 21·34	+5·13	+17 9 35·8	+ 2·3	A. G. Berl. A. 1274
86	4 4 51·18	+1·78	+16 57 15·4	− 3·5	,, ,, 1096
87	4 21 15·15	+3·25	+18 41 7·5	− 5·0	,, ,, 1172
88	8 14 20·26	+2·29	+18 34 4·8	−14·3	,, ,, 3272
89	8 11 21·72	+2·26	+18 48 14·0	−14·1	Berlin 16 (Struve) 842
90	8 11 25·83	+2·06	+18 56 11·5	−13·4	A. G. Berl. A. 3257
91	21 40 14·10	+4·13	−19 42 48·0	+23·9	Cape Durchm. 8078 *
92	21 42 2·05	+3·96	−18 56 40·4	+22·6	Mü.$_1$ 29386 ; Mü.$_2$ 11954
93	21 55 50·87	+3·63	−16 46 39·1	+22·0	A. G. Wash. 8244
94	2 14 21·21	+4·52	+ 4 0 43·1	+19·3	,, Alb. 649
95	2 8 52·43	+4·51	+ 3 54 42·8	+19·6	,, ,, 619
96	15 9 19·42	+2·92	+ 5 6 1·2	− 0·5	Toul. ph. 114, No. 19
97	2 59 39·56	+4·84	+21 9 38·7	+13·9	A. G. Berl. B. 891
98	2 54 38·06	+4·87	+21 1 16·4	+14·6	,, ,, 869
99	2 58 28·57	+4·88	+20 51 35·4	+14·2	,, ,, 888
100	2 53 59·01	+4·91	+20 55 8·4	+14·9	,, ,, 865
101	2 50 31·58	+4·94	+20 28 32·6	+15·5	,, ,, 855
102	2 49 44·62	+4·99	+20 14 25·8	+15·9	,, ,, A. 780
103	2 32 16·55	+5·13	+19 6 58·2	+18·5	Pa. 1890; Pa. ph. †
104	2 28 1·99	+4·98	+18 33 36·1	+18·9	A. G. Berl. A. 695
105	2 36 47·52	+0·93	+17 51 28·9	+ 1·3	Bord. ph. 327, No. 48
106	22 35 52·77	+4·01	− 2 16 38·3	+28·0	Alg. ph. 1381, No. 197
107	22 33 31·10	+4·00	− 2 28 48·6	+28·0	,, 1381, No. 173
108	12 58 30·62	+2·58	− 4 33 43·7	−13·9	Astr. Nachr. 2985, No. 147
109	12 54 50·04	+2·60	− 3 30 22·4	−14·2	A. G. Strb. 4725
110	12 51 12·59	+2·60	− 3 10 44·1	−14·3	,, ,, 4707
111	13 45 43·01	+2·48	+ 0 17 3·0	−11·1	,, Nic. 3644
112	13 42 5·31	+2·50	+ 0 44 34·7	−12·1	,, ,, 3630
113	13 34 32·15	+2·58	+ 1 52 22·7	−12·1	,, Alb. 4767
114	13 26 30·55	+2·60	+ 3 20 3·0	−12·5	,, ,, 4737
115	2 1 20·57	+4·82	+17 16 7·9	+21·2	,, Berl. A. 593
116	1 59 11·22	+4·80	+16 49 8·0	+21·4	,, ,, 586
117	13 43 53·51	+2·61	− 7 37 41·6	−11·4	,, Ott. 4918
118	13 38 49·43	+2·66	− 6 29 24·3	−11·9	Warsch. 3346
119	13 34 10·90	+2·68	− 6 23 46·0	−12·2	A. G. Strb. 4900
120	1 9 36·66	+4·35	+ 6 27 28·9	+24·3	,, Lpz.$_2$ 434

* Approximate position.
† Position obtained through the courtesy of the Director of the Paris Observatory.

Comparison Stars—continued.

No.	Mean R.A.	Red. to Date.	Mean Decl.	Red. to Date.	Reference.
	h m s	s	° ′ ″	″	
121	8 17 47·63	+2·23	+14 15 1·2	−15·1	Bord. ph. 261, No. 94
122	8 17 3·70	+2·08	+15 14 16·2	−14·6	A. G. Berl. A. 3294
123	23 13 49·54	+4·11	− 3 27 47·5	+27·7	,, Strb. 8019
124	23 12 5·35	+4·04	− 3 34 39·7	+27·5	,, ,, 8012
125	23 5 29·19	+3·78	− 4 23 46·0	+26·1	,, ,, 7987
126	0 15 28·04	+4·25	+ 4 48 16·4	+27·1	Toul. ph. 2, No. 137
127	19 42 47·09	+3·38	−10 15 11·2	+19·0	A. G. Cbr. Mass. 6950
128	19 50 7·43	+3·31	−10 19 16·9	+19·3	,, ,, 7002
129	0 12 33·34	+4·21	+ 1 24 19·1	+27·1	A. G. Alb. 38, A. G. Nic. 30
130	0 3 34·15	+4·05	+ 0 38 58·6	+26·3	,, ,, 6
131	21 32 14·69	+3·88	+ 5 46 44·6	+28·3	,, Lpz.$_2$ 10825
132	1 49 22·18	+4·45	+ 2 52 43·9	+21·3	,, Alb. 533
133	1 49 24·68	+4·45	+ 2 47 34·0	+21·2	,, ,, 534
134	1 32 7·86	+4·42	+ 3 54 20·1	+22·3	,, ,, 450
135	11 23 31·54	+2·53	−11 59 57·9	−17·6	,, Cbr. Mass. 4287
136	9 49 30·31	+2·30	+19 58 1·3	−15·2	,, Berlin B. 3865
137	22 44 32·41	+4·02	− 2 34 24·7	+28·0	,, Strb. 7900
138	22 41 11·08	+4·02	− 2 40 9·0	+28·0	,, ,, 7887
139	7 27 3·04	+2·03	+ 3 1 51·4	−16·4	,, Alb. 2870
140	0 23 3·24	+4·09	−20 57 14·5	+24·9	Arg. W. 166
141	0 22 51·62	+4·09	−21 6 24·9	+24·8	,, ,, 164
142	0 14 14·01	+4·07	−21 34 57·0	+24·1	Arg. W. 92; Arg. Gen. Cat. 205
143	0 9 29·02	+4·19	+ 0 1 42·7	+26·8	Alg. ph. 1566, No. 83
144	23 22 42·93	+4·12	+ 1 15 20·9	+28·3	A. G. Nic. 5824
145	1 48 52·49	+4·25	− 8 13 9·7	+18·5	,, Ott. 389
146	1 48 42·20	+4·24	− 8 3 25·7	+18·2	,, ,, 388
147	1 45 55·10	+4·22	− 7 42 15·0	+18·1	,, ,, 376
148	2 24 47·66	+4·27	− 9 6 36·5	+16·9	,, ,, 549
149	2 17 2·96	+4·29	− 9 5 58·9	+16·2	,, ,, 511
150	0 22 2·34	+4·50	+19 36 34·6	+27·5	,, Berl. A. 97
151	0 21 11·77	+4·49	+19 27 47·4	+27·6	,, ,, 90

The Third Satellite of Jupiter. By Rev. T. E. R. Phillips, M.A.

Numerous observations by various observers show that Ganymede (J III.) often presents an elliptical shape, and that the position-angle of the major axis is not always in the same direction relatively to the plane of the orbit. It has been supposed that both the ellipticity and the changes in position-angle are attributable to spots and shadings on its surface, but definitive results as to the rotation period and allied phenomena do not seem to have been deduced. As a matter of fact, while a period at least approximately equal to that of the orbital revolution has been favoured by some observers, notably by Innes (*M.N.*, 70, 44), Campbell and Schæberle (*Ast. Soc. Pacific*, 3, 359), and the late Professor Lowell (*Annals of the Lowell Observatory*, 2, 172), others believe that the period is a short one.

At various times in recent years, without reference to what has been done elsewhere and without noting the position of the satellite in its orbit (so as to avoid any possible effect of bias), I have made a number of observations of the position-angle of the apparent major axis by means of micrometer settings with the 8-inch refractor lent to me by the Society, and during the last apparition (1921) have also secured a number of estimates of the amount of the ellipticity by the use of a scale similar to that given in vol. ii. of the *Annals of the Lowell Observatory*, p. 141, which was kindly supplied by Professor W. H. Pickering.

The observations relating to this last apparition (1921) are given in Table I. In column 2 the times have been expressed as angles in the orbit reckoned from superior geocentric conjunction, and the position-angles as given in column 3 are measured from the axis of Jupiter, the plane of the orbit being assumed the same as that of the Jovian equator. The observations were made on every night when the seeing was sufficiently good—42 nights in all—and, except when changes were observed, means were taken of the separate sets of observations on the same night. The power employed was usually 350, but on special occasions 670 was found useful.

TABLE I.

Date.	Angle from Sup. Geocentric ☌.	Position-Angle of Major Axis (corrected).	Ellipticity.	Sets of Obns.	Seeing. 10 = erfect.	Class.	Position-Angle. O − Curve.
1921. d h m	°	°					°
Jan. 14 14 50	30	322	1·20	1	...	...	−16
Feb. 19 11 20	37	337	1·04	3	4–5	2	− 1
21 11 16	138	323	1·05	2	3–4	2	−19
24 9 52	286	...	round	1	6	1	
26 12 10	32	350	1·05	2	7	1	+12
Mar. 1 12 30	184	28	...	1	2–3	3	+ 9
2 11 0	231	...	round	2	4	2	
7 10 30	123	328	1·10	1	8	1	13
8 13 58	180	22	...	2	5	2	

TABLE I.—*continued.*

Date.				Angle from Sup. Geocentric δ.	Position-Angle of Major Axis (corrected).	Ellipticity.	Sets of Obns.	Seeing. 10 = Perfect.	Class.	Position-Angle. O – Curve.
1921.	d	h	m	°	°					°
Mar.	9	10	52	224	28	1·10	3	6–5	1	+ 3
	9	13	5	229	...	round	1	7–8	1	
	11	12	10	328	352	1·02 ?	1	5	2	+13
	14	11	40	118	359	1·06	1	4	2	+18
	27	10	0	50	322	1·10	1	7	1	−17
	31	8	45	249	...	round	1	7	1	
Apr.	1	7	43	298	339	1·04	4	6–5	2	− 1
	1	11	55	306	326	1·10	2	6–5	1	−14
	2	11	38	356	338	1·04	1	7–8	1	0
	3	9	15	42	327	1·11	2	5–4	2	−11
	4	8	45	91	345	1·08	2	5–4	2	+ 5
	4	12	18	99	326	1·10	1	4	2	−14
	6	10	58	192	30	1·12	2	6–7	1	+ 6
	7	9	0	243	...	round	1	4	3	
	12	11	34	141	331	1·08	2	6	1	−12
	14	9	40	237	...	round	1	6	1	
	16	7	35	333	332	1·08	1	3	3	− 6
	18	10	7	79	359	1·12	2	7–8	1	+19
	19	8	42	127	336	1·08	1	5	2	− 6
	19	10	48	131	345	1·08	1	5	2	+ 3
	23	10	30	332	330	1·06	1	6	1	− 7
	24	8	20	18	345	1·16	1	8–9	1	+ 7
May	8	10	0	6	326	1·16	1	6	1	−12
	10	10	18	107	340	1·16	2	7	1	0
	12	9	13	205	21	1·04	1	7	1	− 6
	14	10	45	309	...	round	1	5	2	
	17	9	15	97	342	1·10	1	5	2	+ 2
	19	8	50	197	18	1·10	3	5	1	− 7
	20	9	17	247	...	round	3	5	1	
	21	10	0	300	...	round	1	4	3	
	22	9	55	350	351	1·06	1	5	2	+13
	28	8	20	288	...	round	1	5	2	
June	1	8	38	129	351	1·04	1	4–5	2	+10
	2	8	55	180	10	...	1	3–4	3	− 5
	10	9	30	223	...	round	1	5–6	1–2	
	14	8	30	62	336	...	1	6	1	− 3

The figures in columns 2 and 3 have been plotted on the accompanying diagram (fig. 1), with the position-angles as ordinates and the angles in the orbit as abscissæ. The curve was drawn from the 1921 observations only, and the figures in the last column are the deviations of the observed position-angles from this curve.

The mean of the residuals ($\pm 9°$) and the probable error of an observation ($\pm 7°$) are perhaps as small as might be expected from the nature of the quantities to be observed.

The observations of position-angle obtained in previous years have also been plotted on the same diagram, and it will be seen that they agree generally with the 1921 results. The position-angles of 1915 show consistently larger values, but there is no certain difference in the phase. These earlier observations are given in Table II. and compared with the 1921 curve in the last column.

TABLE II.

Date.	Angle from Sup. Geocentric ☌.	Position-Angle of Major Axis (corrected).	Position-Angle, O − C Curve (1921).
d h m	°	°	°
1915 Aug. 16 13 30	30	346	+ 8
19 12 40	180	48	+33
23 12 0	20	347	+ 9
25 12 0	121	6	+25
27 12 30	223	34	+ 9
29 12 0	323	round	
Sept. 4 11 25	270	round	
7 12 50	59	4	+25
17 13 0	203	44	+17
Oct. 3 11 15	287	round	
13 7 30	63	357	+18
14 11 0	121	3	+22
Nov. 2 10 0	356	37	+57
1917 Oct. 1 17 45	182	25	+ 7
Nov. 13 11 10	175	20	+12
Dec. 19 7 20	183	13	− 5
1918 Jan. 27 9 20	352	353	+15
31 5 30	185	19	− 1
Nov. 21 9 55	184	9	−11
1919 Jan. 17 9 10	177	13	+ 1
17 12 35	184	8	−12

An inspection of the diagram shows that the position-angle of the major axis changes in a manner that is definitely related to the place of the satellite in its orbit, reckoning from superior geocentric conjunction. Its N. extremity is west (P.A. = 340° ±) of the pole of the orbit from 0° to 140°, east of it (P.A. = 10° to 30° ±) from 175° to 225°, and, after an interval of at least 75° during which the satellite appears quite round, returns to its original position. Unfortunately no observations have been made between 140° and 175°, where the change from the west to the east position seems to take place with considerable suddenness.

Turning now to the estimates of ellipticity (Table I., column 4), it will be found that there are considerable differences, and they are not

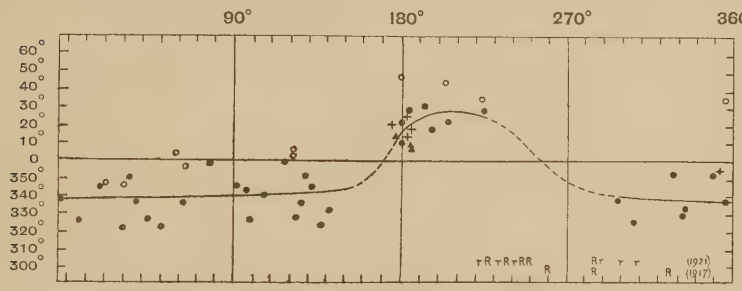


FIG. 1.—Position-Angles of Major Axis.

● = Observations made in 1921.
⊙ = ,, ,, 1915.
\+ = ,, ,, 1917–18.
▲ = ,, ,, 1918–19.

The letters R and r show the angles where the satellite was observed to be round, the capitals denoting the best observations.

always as consistent *inter se* as one could wish, but, grouping them about the angles 0°, 30°, 60°, etc., we get the following mean values, the subscript figures denoting the number of observations and those in brackets the mean deviations :—

$$1{\cdot}09_3(\cdot05),\ 1{\cdot}11_5(\cdot05),\ 1{\cdot}10_1,\ 1{\cdot}10_4(\cdot01),\ 1{\cdot}09_6(\cdot03),\ 1{\cdot}07_2(\cdot02),\ 1{\cdot}12_1,$$
$$1{\cdot}06_4(\cdot04),\ 1{\cdot}00_6(\cdot00),\ [1{\cdot}00_0],\ 1{\cdot}02_6(\cdot03),\ 1{\cdot}05_3(\cdot02).$$

There is clearly an oscillation in the period of the satellite's variation (see fig. 2), and the second maximum about and following inferior conjunction seems to be real. The ellipticity always appears strongly marked at such times, but estimates were not made during actual transit across the disc, as the polar regions are often invisible against the background of the planet's surface. The analytical expression for the curve to the third harmonic is

$$1{\cdot}07 + {\cdot}05\sin(\theta + 2^\circ) + {\cdot}03\sin(2\theta + 76^\circ) + {\cdot}01\sin(3\theta + 243^\circ).$$

The natural inference from the 1921 observations is that the periods of the satellite's rotation and revolution are very closely similar, and the fact that there has been no appreciable change of phase in a period of six years appears to indicate that the agreement is substantially exact.

To explain the changes of position-angle and ellipticity above described, it seems natural to suppose that they arise from dark regions on the satellite's surface which are variously presented in the course of a revolution. It has not been possible to make a systematic study of such markings at Headley, but they have been recorded occasionally, especially near the times of inferior conjunction. On such occasions a broad dark band has been several times seen extending across the disc at right angles to the major axis, having its parts near the extremities of the latter bright, and other features have been occasionally

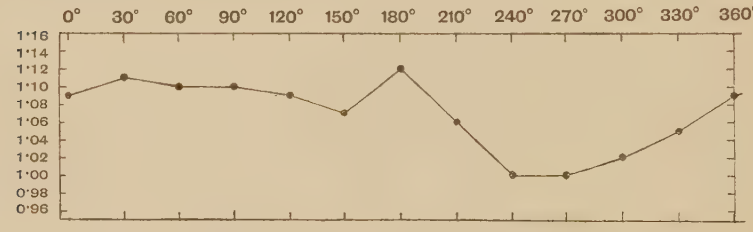


FIG. 2.—Estimates of Ellipticity (the equatorial axis = 1·00).

glimpsed—especially a shading at the limb giving the satellite a gibbous rather than an elliptical appearance. The account of a remarkable transit of J III. observed by Dr. W. H. Steavenson and the writer on 1917 October 1 is published in *B.A.A. Journal*, 28, 56, and it seems to afford a striking illustration of how the relative brightness of different parts of the disc may affect the apparent shape of the satellite.

Measures of the diameters of the satellites by the interferometer method were made by Michelson at Mount Hamilton in 1891 (*Pub. Ast. Soc. Pacific*, 3, 274), and by Hamy at the Paris Observatory in 1898 (*C.R.*, 128, 583), but the published accounts do not state whether the position-angle of the slits was varied, or, if so, whether any differences were found in J III. which give support to the apparent ellipticities visually observed. It would be of much interest if this work were repeated, and observations made when the satellite is in different parts of its orbit.

Headley:
1921 *December* 3.

On the Law of Rotation of the Sun. By H. F. Newall, F.R.S.

In the present note I desire to call attention to certain implications of the law of rotation, with which the Sun appears to try to conform, if we may judge the matter in the light of spectroscopic determinations.

When Carrington first formulated an empirical expression for the daily sidereal angular motion of sunspots in different latitudes about an axis of rotation and gave it the form

$$\xi = A - B \sin^{\frac{1}{4}} \phi,$$

Babinet pointed out (*Comptes Rendus,* 59, 481) that for the sake of symmetry in the two hemispheres the power index of $\sin \phi$ should be an even integer; and Faye emphasised and developed this thesis.

Spectroscopists have sought in recent years to convince themselves that their observations conform satisfactorily with the allied relation

$$v + v_1 = (a - b \sin^2 \phi) \cos \phi \quad . \quad . \quad . \quad (A)$$

where v is the velocity in the line of sight deduced from their actual observations; v_1 is a correction allowing for the orbital motion of the earth so as to convert synodic periods into sidereal; ϕ is the heliographic latitude; and a and b are adjustable velocities which have hitherto been regarded as constant from equator to pole. The question of the constancy of a and b *in time* throughout the eleven-year cycle has been freely discussed, but the evidence has hitherto been so conflicting, that it has not been possible to deduce any law of consistent variation in time.

The considerations which I now wish to set forth lead to a conclusion which I think deserves careful discussion. It is based on the idea that we have no reason to expect from a sun in tumultuous motion any great consistency in spectroscopic determinations. We should rather expect departures from a law of the kind implied in relation (A) than be surprised at them. If such departures had not been found, it would mean that the reversing layer must extend into regions either outwards, where turbulent motions cease and equilibrium is maintained by radiation and diffusion rather than by convective movements: or else inwards, where viscosity might be expected to preclude the possibility of violent motions.

I have been engaged for some years past in studying the outbursts and motions of sunspots as recorded in the Greenwich Photoheliographic Results, in the hope of being able to detect indications of local disturbances in what may be called the latitude-gradients of the daily sidereal motions of spots and spot-groups, $\frac{d\xi}{d\phi}$. It has been a troublesome research, but I hope to be able to present the very complex data in intelligible form before long; and this hope is strengthened by my having found helpful considerations suggested by a study of this relation (A)

$$v + v_1 = (a - b \sin^2 \phi) \cos \phi ;$$

and being desirous of setting forth these considerations prior to the meeting of the Astronomical Union in Rome in April next, I confine my attention in the present note to the spectroscopic problem.

The latitude-gradient of the velocity is

$$\frac{d(v+v_1)}{d\phi} = -\sin\phi(a+2b-3b\sin^2\phi).$$

It will be convenient to regard this as a positive retardation. If we inquire whether there can be a maximum in the value of the gradient in any latitude, we find

$$\frac{d^2(v+v_1)}{d\phi^2} = -\cos\phi(a+2b-9b\sin^2\phi);$$

and equating to zero, we get

$$\text{either } \phi = 90^\circ, \text{ or } \sin^2\phi = \frac{a+2b}{9b};$$

and inasmuch as the latter equation indicates the possibility of maxima depending on the relative values of a and b, we investigate it further, and find that it gives maxima in the sense that as we pass poleward the relative retardation of contiguous zones of latitude increases to a maximum at a latitude ϕ which depends largely on the value of b.

Now it appears from observation that a is fairly constant; and so we pay attention to b. We have for a maximum in the retardation

$$b = \frac{a}{9\sin^2\phi - 2}.$$

So long as we may regard a as having a value which varies, if at all, only within narrow limits, it is clear that we shall only get maxima in the values of $\frac{d(v+v_1)}{d\phi}$ if b has appropriate values. Table I. shows the values which b must have to give maxima in latitudes 90°, 80°, 70°, etc., consistent with equatorial velocities $a = 2{\cdot}10$ km./sec., $a = 1{\cdot}95$, and $a = 1{\cdot}80$.

TABLE I.

ϕ.	$b=\frac{a(=2{\cdot}10)}{9\sin^2\phi-2}$.	$b=\frac{a(=1{\cdot}95)}{9\sin^2\phi-2}$.	$b=\frac{a(=1{\cdot}80)}{9\sin^2\phi-a}$.
Pole 90°	·300 km./sec.	·278	·257
80	·312	·290	·267
70	·353	·328	·303
60	·440	·410	·379
50	·640	·594	·549
40	1·22	1·12	1·05
30	8·40	7·8	7·20
28·1	∞	∞	∞
20	neg.	neg.	neg.
10	,,	,,	,,
Equator 0	,,	,,	,,

We see, for instance, that if $b = \cdot 640$, there will be a maximum of retardation at $\phi = 50^\circ$, if $a = 2 \cdot 10$; or at lower values of ϕ if a is smaller than that value.

It is clear that if we release ourselves from the limitations involved in accepting only such numerical values of the constants a and b in the relation (A) as are required in the solar problem, then we should find that in order to get a maximum at latitudes lower than 40°, appropriate values of a and b can be assigned, including the passage of b through increasing finite values to infinity at $\phi = 28^\circ \cdot 1$ ($\sin^2 \phi = \frac{2}{3}$). But these are of no interest in the solar problem.

We must remember that the relation (A) is an empirical expression accepted as expressing fairly closely the distribution of spectroscopic velocities in latitude; but it has not been based on any physical or hydrodynamical foundation, so far as I am aware. Babinet's even power of $\sin \phi$ is the only physical point put into it. Hence a careful scrutiny of the implications of the empirical law seemed desirable; and it has resulted in the finding of a clue which seems as if it may lead to a better understanding of some of the conditions underlying the circulation of the Sun's outer strata, and may serve to throw some light upon the relegation of sunspots to certain zones of latitude, and also possibly upon the latitude-movements of the spots and the prominences during the eleven-year cycle.

Though, for the solar problem, I have utilised the numerical mean values of a and b recently found by spectroscopists, in order to deduce the *position* of the maximum, and have in my lectures for some years past pointed out that these values place the maximum between latitudes 50° and 60°, I had not investigated the matter in detail.

But finding recently that Halm's observations in 1901 agreed well with Dunér's for that year in assigning the high values 0·70 and 0·79 to b, and further noting that the year 1901 saw the outburst of sunspots in high latitudes close upon 40° after the marked sunspot minimum, during which Dunér's observations had shown considerably smaller values of b, it occurred to me to revert to the matter; and Table I. is one of the first results. It seems to indicate that values of b higher than those commonly found by spectroscopists might possibly be forthcoming only at certain epochs in the sunspot cycle. And further examination of the observations supports the interesting conclusion that probably the values of both a and b are functions of the epoch in the sunspot cycle.

Spectroscopic observations of the solar rotation show that a has values generally well within the limits 2·15 to 1·85 km./sec., and that b has values ranging from about 0·300 to about 0·800 km./sec. and seldom approaches the value 1 km./sec. Thus we must infer from Table I. that the lowest latitude in which we can hope to see an organised maximum in the value of the gradient is about 40°, with a value b equal to about 0·6 or 0·8. We have to remember, however, that $d(v + v_1)/d\phi$ as here derived is the gradient of velocity for the gases on the spherical surface of the Sun, and as such it must be regarded as consisting of two parts, one arising from purely geometrical considerations due to the spherical shape, and the other arising from relative

motion of the gases on a sphere of reference. The choice of the proper equatorial velocity for a rotating *rigid* sphere of reference is dealt with in a later paragraph. It is enough here to state that maxima related to the maxima indicated in Table I. survive as maxima of retardation to be reckoned with. The fact that we do not usually find high values of b must be interpreted as indicating that there is something in the solar processes which forbids the corresponding distributions of velocities in contiguous zones of latitude between the equator and the latitude 40°.

We can, in fact, hardly resist the conclusion that with high values of b and of $-d(v+v_1)/d\phi$ the motion must change from stream-line motion to turbulent motion; and when the turbulence is organised (possibly by some form of precipitation or outburst), spots and vortices burst out near the latitude 40°.

When turbulence sets in, mixing of the contiguous zones at the latitude (say 40°) of the maximum gradient takes place with a reduction of the mean velocity. There follows a necessary increase of difference of velocity relative to that in the contiguous zone, say at latitude 39°, and a consequent increase of $d(v+v_1)/d\phi$ at 39°; and so the process of outburst of turbulence, accompanied here and there by spots and vortices, is gradually propagated towards the equator. On the other hand, in latitudes higher than 40° the relative motion of contiguous zones of latitude is diminished, and this tends to the persistence of stability polewards, in contradistinction to the propagation of instability equatorwards. Thus it would appear that there must be a gradual change (*in time*) of the gradient of velocity of the vapours, a change which is slowly propagated from the poles towards the equator, culminating after sunspot minimum in an outburst of sunspots about latitude 40°, with the subsequent convergence of the wide sunspot zones towards the equator in the eleven-year cycle. There is evidence that, when the spots have died out near the equator, the equatorial velocity increases; and this increase probably contributes to the width of the zone of instability between the equator and latitude 40°. Clearly a circulation between polar regions and equatorial regions involves movement of moment of momentum with very different effects, not only according to the direction in which it takes place, but also according to the level of the stratum in which the moment of momentum is carried.

This conception is, I take it, essentially that of Faye; but I am not aware that it has been elaborated in any detail in regard to the data now made available by the wealth of spectroscopic observations accumulated in the last twenty years. Before this was available, the problem of the solar rotation had been attacked by Harzer (*Ast. Nach.*, **127**, 17) and by Wilsing (*A.N.*, **127**, 333) and by Sampson (*Mem. R.A.S.*, **51**, 123) and by Wilczynsky (*Ap. J.*, **4**, 101; **7**, 124). Their work proceeds, however, chiefly in the study of the internal structure of the Sun; and it was impeded by uncertainty as to the gradients of density, pressure, temperature, and viscosity as we pass inwards into the body of the Sun. The effort of the present note is to extract the maximum amount of information from the mass of observational data relative to the actual motions of the vapours near the reversing layer. I proceed first to give some tables, which serve to show schematic velocities and gradients.

Table II. exhibits the values of $v + v_1$ deduced for different values of b with an assumed value of a, viz. 2·10 km./sec.

Table II.

Velocities satisfying Relation (A), with $a = 2{\cdot}10$ km./sec. and Various Values of b.

b. / ϕ.	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	2·100	2·100	2·100	2·100	2·100	2·100	2·100	2·100
10	2·069	2·062	2·061	2·059	2·058	2·055	2·049	2·037
20	1·974	1·952	1·947	1·941	1·935	1·925	1·904	1·858
30	1·819	1·775	1·760	1·753	1·742	1·723	1·680	1·590
40	1·609	1·545	1·530	1·513	1·497	1·470	1·405	1·275 §
50	1·350	1·275	1·256	1·234	1·222	1·183	1·112 §	0·952
60	1·050	0·975	0·956	0·938	0·918	0·885 §	0·810	·656
70	0·718	·657	·643	·629	·612 §	·585	·523	·400
80	0·365	·332	·323	·315	·306	·291	·258	·180
90	0	0	0	0	0	0	0	0

§ Points of inflexion.

The column headed $b = 0{\cdot}00$ gives the velocities for a solid or rigid sphere, which has an equatorial velocity of $a = 2{\cdot}10$ km./sec. The tendency of these velocities with varying values of b is best seen in the graphs plotted in fig. 1. For low values of b, between 0·000 and 0·300, the latitude-velocity curves are concave towards the origin of co-ordinates. For higher values of b, points of inflexion appear in the curves. The positions of these points are indicated by § in Table II.

The inflected spindle or family of curves seen in fig. 1 shows the limits of the relation (A) for constant equatorial velocity of the vapours ($a = 2{\cdot}10$) and various values of b. If both a and b vary at different epochs whilst they may be held to be constant from pole to equator at any one epoch, then the corresponding inflected spindles will move slightly on the diagram, with the polar end fixed at $\phi = 90°$, and the equatorial end moving through appropriate distances on the axis of velocities.

Now when we compare the spindles corresponding with $a = 2{\cdot}10$ and $a = 1{\cdot}90$, we see that between the latitudes 90° and 40° the curve ($a = 2{\cdot}10$, $b = 1{\cdot}00$) can scarcely be distinguished from the curve ($a = 1{\cdot}90$, $b = 0{\cdot}800$). There is no doubt about distinguishing between the equatorial ends of such curves.

We shall see in a later paragraph some justification for treating the observational data in at any rate two parts, one above latitude 40°, the other below that value. We shall see that the polar regions give velocities which are better fitted by the relation (A) with values of a and b different from those which best satisfy the equatorial regions

—a result which naturally follows from using four constants instead of two. [This procedure is an alternative to that sometimes adopted of inserting an additional (b, ϕ) term in the bracket of relation (A).] We need not lay undue stress on the extrapolated values of a derived from the data of the polar region.

The implication is that the circulation of the reversing vapours

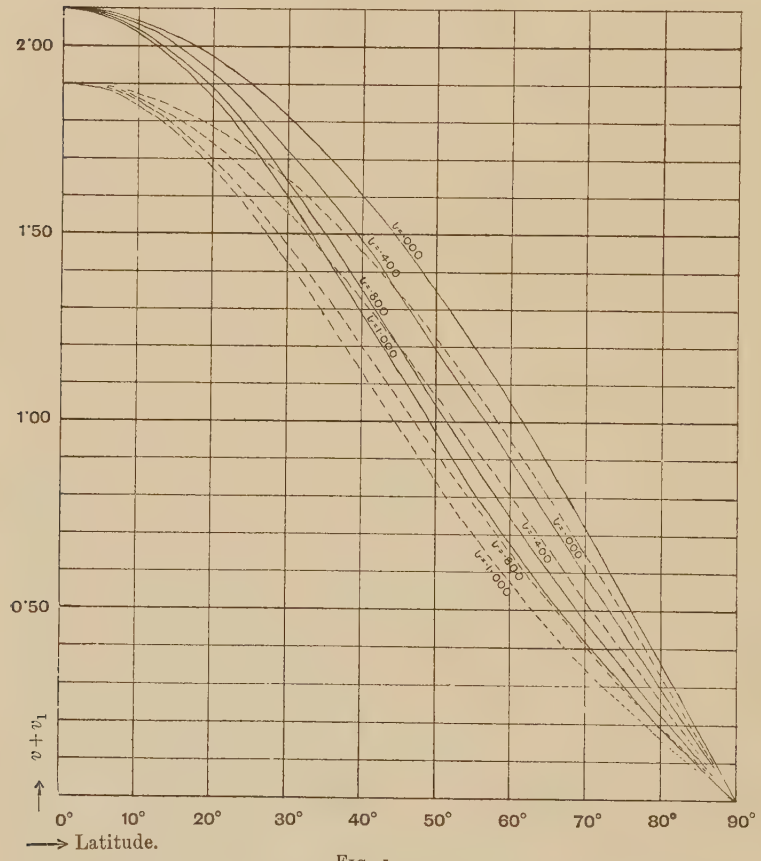


FIG. 1.

probably involves a movement of moment of momentum between the polar and equatorial regions: and evidence is adduced below to show that the velocities a and b are most probably functions of the phase of the eleven-year cycle of sunspot frequency.

Table III. exhibits the values of $-\frac{d(v+v_1)}{d\phi}$, the latitude-gradients of velocity in km./sec. per radian, for an assumed value of a, viz. 2·10 km./sec., and different values of b.

TABLE III.

Latitude-gradients of Velocity (Negative Sign omitted).

$b./\phi.$	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
10	·365	·432	·448	·465	·482	·512	·578	·714
20	·718	·820	·859	·887	·916	·966	1·079	1·310
30	1·050	1·175	1·206	1·238	1·268	1·325	1·450	1·706
40	1·350	1·448	1·472	1·497	1·521	1·566	1·663	1·864*
50	1·609	1·645	1·655	1·664	1·673	1·689	1·726*	1·801
60	1·819	1·775	1·764	1·754	1·742	1·723*	1·680	1·591
70	1·974	1·852	1·821	1·790	1·760*	1·704	1·582	1·330
80	2·069	1·889	1·849	1·799	1·754	1·675	1·495	1·127
90	2·100	1·900	1·850	1·800	1·750	1·660	1·460	1·050

The asterisk * indicates maxima.
[For graphs of some of the figures in the table, see fig. 2 below.]

Table IV. exhibits the values of the gradients $-\frac{d(v+v_1)}{d\phi}$ expressed in metres per second per degree of latitude.

TABLE IV.

Latitude-gradients of Velocity (Metres per Second per Degree).

$b./\phi.$	0·00.	0·200.	0·250.	0·300.	0·350.	0·440.	0·640.	1·050.
0°	0	0	0	0	0	0	0	0
10	6·4	7·5	7·8	8·1	8·4	8·9	10·1	12·4
20	12·5	14·3	15·0	15·5	16·0	16·9	18·8	22·9
30	18·3	20·5	21·0	21·6	22·1	23·1	25·3	29·8
40	23·5	25·3	25·7	26·1	26·6	27·3	29·1	32·5*
50	28·1	28·7	28·9	29·1	29·2	29·5	30·1*	31·5
60	31·7	30·9	30·8	30·6	30·4	30·1*	29·3	27·8
70	34·5	32·3	31·8	31·2	30·7*	29·7	27·6	23·2
80	36·1	33·0	32·3	31·4	30 6	29·2	26·1	19·7
90	36·6	33·2	32·3	31·4	30·5	29·0	25·5	18·3

* Denotes a maximum.

The relation (A) gives the observed velocities of the vapours at different latitudes. It would be easy to compare these velocities with those on a rigid rotating sphere, if only we knew what angular velocity to give it. It seems natural to have recourse to some internal core with a large store of angular momentum as the obvious sphere of reference, from which we should deduce the latitude-changes in velocity of the vapours. The maintenance of the observed fairly constant velocity a

at the equator seems to demand some such store of angular momentum, unless we are to regard it as brought about by a struggle between an accelerator and a retarder, one inside, the other outside, the layers of vapours. In any case, it is clear that in dealing with the gradient $d(v+v_1)/d\phi$ we must be careful to discriminate between the geometrical part due to the spherical shape of the Sun and that part which constitutes the departure of the velocities of the vapours from the law of rotation of a rigid sphere.

If we take a rotating rigid nucleus with equatorial velocity A as the sphere of reference, the velocities due to the nucleus are given by $N = A \cos\phi$. The relative velocity of the vapours in latitude ϕ is

$$v + v_1 - N = (a - A - b \sin^2\phi) \cos\phi.$$

This has zero values when $\phi = 90^\circ$ and when

$$\sin^2\phi = \frac{a-A}{b};$$

and if $a = A$, then $\phi = 0^\circ$.

The gradient $\dfrac{d(v+v_1)}{d\phi} = \dfrac{dN}{d\phi}$, when $\sin^2\phi = \dfrac{a-A+2b}{3b}$;

and if $a = A$, $\sin^2\phi = \frac{2}{3}$; consequently $\phi = 54^\circ{\cdot}7$, and it is independent of the value of b.

This value of ϕ is of interest in connection with the distribution of prominences, and in particular with the phenomena of Deslandres' dark circumpolar filaments.

The value adopted for A is of special importance when we seek the proper interpretation of the maximum value of $d(v+v_1)/d\phi$, inasmuch as the part of the latter available for the explanation of the outburst of sunspots depends upon it. It seems probable that A must have a value which is equal to the highest observed value of a; and that smaller values of a may be produced by periodic wandering of vapours towards the equator from higher latitudes.

But it is clear that if we appeal to the maximum value of $\dfrac{d(v+v_1-N)}{d\phi}$ as the source of the production of instability in the relative motions of the gases, we must not overlook the shearing stresses between the vapours and the nucleus. An essential part of my present suggestion as to the origin of sunspots consists in taking account of the shearing stresses at the surface of the cone which has its apex at the Sun's centre and a semi-angle equal to the colatitude. These conical shears may lead to turbulent motion about axes always vertical to the Sun's surface, whatever latitude we may be dealing with.

On the other hand, the stresses introduced by the failure of the vapours to maintain the same velocity as the underlying nucleus lead to turbulence about axes far more nearly tangential to the surface of cylinders about the Sun's axis. These latter stresses are responsible for the vortices that have engaged the attention of Wilczynsky and Emden and others. It may be that the conical shears serve to turn the ends of Emden's vortices more nearly perpendicular to the surface of the Sun.

The values of Table III. are plotted in fig. 2 as a graph of gradients and latitudes. Here again, as in fig. 1, the curve for $a = 2{\cdot}10$ and $b = 0{\cdot}000$ is the graph for a rigid sphere rotating with equatorial velocity 2·10 km./sec. The difference between the ordinates for any given value of b and for $b = 0{\cdot}0$ gives the gradient in different latitudes of the vapours relative to the rigid sphere of reference ($a = 2{\cdot}10$), and we see

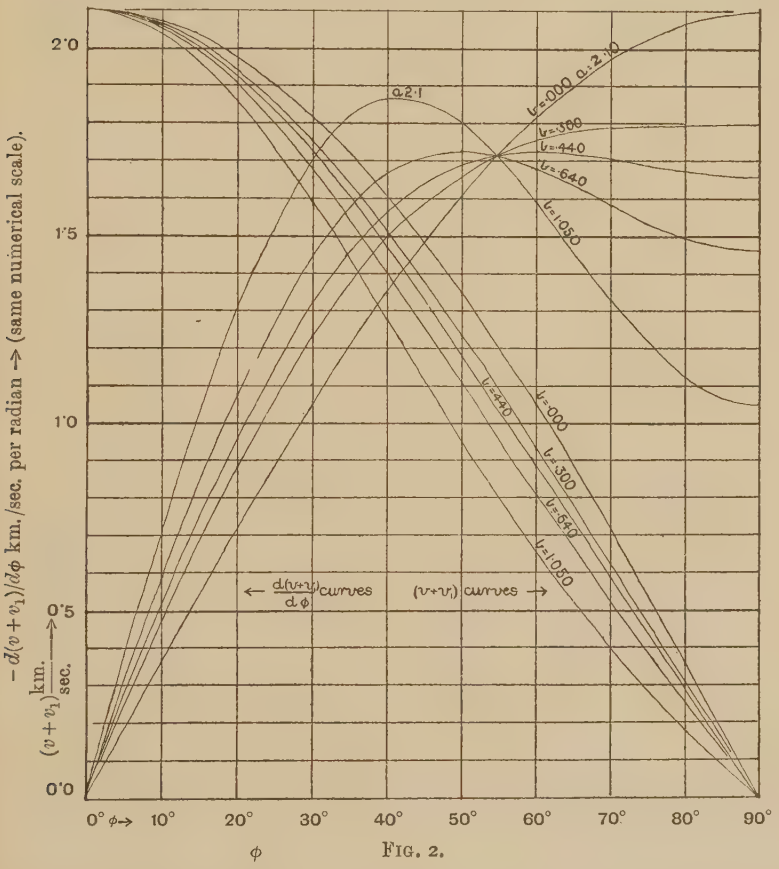


FIG. 2.

that the effective gradient of the velocity of the vapours is of the order of 10 metres per second per degree for the higher values of b.

Now the linear value of a degree on the Sun's apparent surface is about 12,000 km., and a wind of 100 km. per hour travels with a speed of 27·7 metres per second. Hence an observer stationed in latitude 30° on the sphere of reference ($a = 2{\cdot}10$) might find that, with $b = 0{\cdot}640$, the vapours 6000 km. to the south of him would be moving at a rate of about 25 km. per hour relative to the vapours 6000 km. to the north of him. The value of the kinematic viscosity (ν) which

would bring about instability would depend on the critical value of Vd/ν.

The viscosity with which we have here to deal is the kinematic viscosity $\nu = \eta/\rho$ of the vapours in the neighbourhood of the reversing layer, η being the ordinary (Poiseuille) coefficient of viscosity and ρ the density of the vapours. If we assume that the density in the reversing layer is of the order of $\frac{1}{10}$ of the normal density of atmospheric air, and further, that the ordinary viscosity η increases as the square root of the absolute temperature, then the kinematic viscosity in the reversing layer would be

$$\nu = \frac{10\eta_0}{\rho_0}\left(\frac{T}{273}\right)^{\frac{1}{2}},$$

when η_0 and ρ_0 are the values at 0° C. and 760 mm., and T is the absolute temperature of the reversing layer.

Now for hydrogen, $\eta_0 = 0{\cdot}84 \times 10^{-4}$, $\rho_0 = 0{\cdot}90 \times 10^{-4}_{.2}$, so that $\frac{\eta_0}{\rho_0} = 0{\cdot}9 = \nu_0$ and $\nu_H = 10 \times 0{\cdot}9 \times \left(\frac{T}{273}\right)^{\frac{1}{2}}$.

(For mercury vapour, $\eta_0 = 1{\cdot}62 \times 10^{-4}$ at 0° C., if we accept Koch's value extrapolated from observations made at 300° and 380° C. This is apparently the only metallic vapour for which Poiseuille's coefficient has been determined.)

Again, $\left(\frac{T}{273}\right)^{\frac{1}{2}} = 5$ for $T = 6825°$ Abs. C. : or 6 for $T = 9820°$ Abs. C.

Hence for hydrogen in the reversing layer we have $\nu_H = 45$.

(For glycerine, $\eta_0 = 46{\cdot}0$, $\rho_0 = 1{\cdot}26$; $\nu = 36{\cdot}5$ at 0° C.)

These figures indicate a degree of viscidity which, while it argues well for effective rigidity in some layers close to the reversing layer, seems to preclude the possibility of the observed motions of the vapours outside it. The generally accepted view, that the pressure diminishes outwards as in a gravitational field (with $g = 27{,}000$ cm./sec.^{2}), involves a very rapid diminution of density in the vapours outside the reversing layer. Accordingly, the values of ν increase very rapidly from a prohibitive value in the reversing layer to still more prohibitive values outside it; and we are at a loss to explain the possibility of the observed signs of motion in the sunspot zones, the observed rapid motions of the prominences, and the clear indications of turbulent motions disclosed in the spectroheliograms of hydrogen. The observations forbid the rapid increase of ν. The only loophole of escape from the dilemma appears to lie in an admission that $d\rho/dr$ is much smaller than is usually allowed.

Discussion of Recent Spectroscopic Observations.

So far for *schematic* distributions of velocities and gradients. We pass now to a scrutiny of the actual results of spectroscopic observations. After the assiduous labours of Dunér, Halm, W. S. Adams, Plaskett, and many other observers, we have arrived at methods which give values of v of such precision that it is obvious that the observations deserve a confidence which is hardly yet admitted by the observers themselves.

The treatment of the observations usually employed is to accumulate observations of the velocity in the line of sight on many days at different solar latitudes; and, when a sufficient number has been secured, to treat them by the method of least squares for determining the most probable values of a and b in the adopted relation (A). The result is that we have found (unexpected?) variations in those values.

The considerations that I have now to urge tend to show that the secret lies in the fact that a least-square combination of observations at widely separated latitudes into *one* equation of the form of the relation

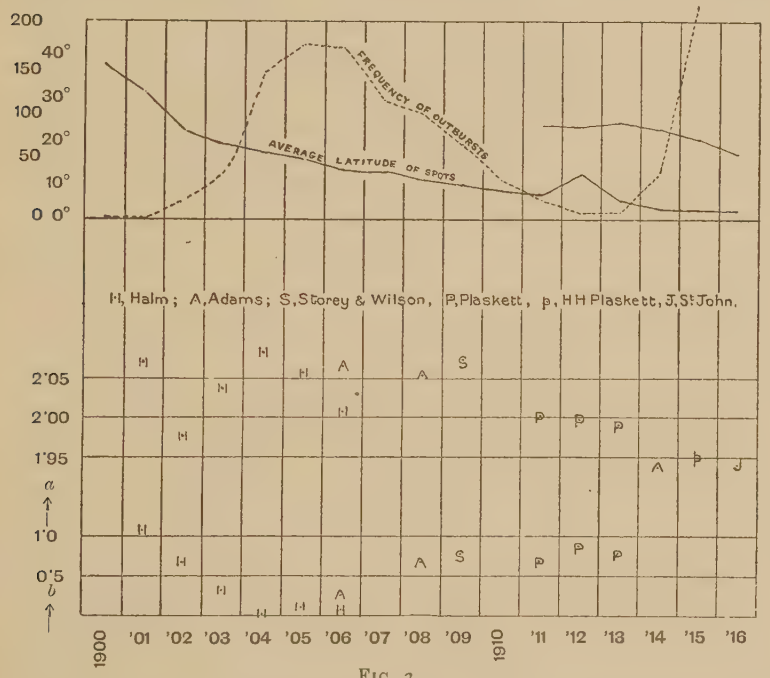


FIG. 3.

(A) is inadequate, by reason of real changes in the values of a and b at different latitudes—changes which are very probably a function of the phase of the eleven-year cycle of frequency of sunspots, or rather a function of the phase of an eleven-year cycle of changes in the whole mode of solar circulation, the sunspots being simply a symptom of this cycle of changes.

I deal with observations for every year in the sunspot cycle 1901–1913, except for the year 1910. Table V. exhibits the results of least-square solutions for each year. I have carried these out, firstly, with a combination of the spectroscopic velocities for all available latitudes into one equation of the form of relation (A); and the most probable values of the constants a and b are set forth in columns 3 and 4; these values are supposed to hold good from equator to pole. [It should be

stated that the observational data in most cases have been gathered by comparisons of the spectroscopic velocities at the limb of the Sun at points at the ends of solar diameters; thus they may include departures from regularity, which may in part be attributable to want of symmetry in the two hemispheres.]

The same observational data have next been divided into two parts, the one relative to latitudes 0° to 40° (equatorial half), the other relating to the latitudes 40° to 90° (polar half). Least-square solutions for a and b have then been carried out for these two parts independently, with the results shown in columns 5, 6, 7, and 8. The residuals (O − C) in nearly every year show (as is natural when four constants are utilised instead of only two) that the observations are better represented by values of a and b restricted to polar and to equatorial regions respectively.

It follows that the value of a deduced from the equatorial region approximates generally more closely to the observed equatorial velocity than the unrestricted value of a; and, on the other hand, the value of a deduced from the polar region is a somewhat wild extrapolation. This is indicated by enclosure in brackets in column 7.

The figures of Table V. are plotted in fig. 3 in graphs of a and time and b and time.

TABLE V.

Values of a and b derived by Weighted Least-square Solutions.

		All Latitudes.		Equatorial half, 0° to 40°.		Polar half, 40° to 90°.	
		a.	b.	a.	b.	a.	b.
1901	Halm	2·06	·70				
	N.	2·06	·74	2·07	·46	(2·40)	1·18
1902	Halm	1·973	·560				
	N.	1·975	·584	1·976	·518	(2·056)	·684
1903	Halm	2·036	·251				
	N.	2·041	·272	2·036	·264	(1·968)	·318
1904	Halm	2·075	·271				
	N.	2·054	·184	2·082	·337	(1·921)	·010
1905	Halm	2·039	·245				
	N.	2·027	·211	2·056	·392	(1·949)	·103
1906	Halm	2·010	·294				
	N.	2·006	·276	2·007	·199	(1·845)	·073
1907	Adams	2·055	·480				
	N.	...	...	2·065	·559	(1·919)	·262
1908	Adams	2·053	·546				
	N.	...	...	2·054	·592	(2·156)	·693
1909	Storey & Wilson	...	...				
1910	N.	2·088	·496	2·074	·433	(2·237)	·792
1911	Plaskett	2·012	·500				
	N	...	...	2·003	·415	(2·156)	·673
1912	Plaskett	2·002	·558				
	N.	...	...	1·999	·529	(2·210)	·870
1913	Plaskett	1·993	·518				
	N.	...	...	1·991	·479	(2·161)	·788

I have hesitated to draw a mean curve through the values of a and of b, for I have had some doubt as to whether the observations of Halm which deal with the Dunér lines at 6302 are quite comparable with the observations of Adams and Plaskett which relate to many more lines in more extended regions in the solar spectrum. I regret that I have not been able to find published accounts of detailed observations for the war-years 1914–1919; it is a pity that these are lacking as yet, but when they are available it will be interesting to see whether the indications of the graphs of fig. 3 are confirmed by observations which cover the whole of an eleven-year cycle. It may be that we have then to exclude the earlier values till 1906, and to lay greater stress on the more modern observations, which constitute more homogeneous material.

I must allow that the arguments adduced above partake of the nature of groping for light in a somewhat disappointingly obscure subject. If, however, we are to lay any weight on the considerations which I set forth, it would appear that the relation (A) should be replaced by an expression of the form

$$v + v_1 = \left[a_0 \left\{ 1 + \frac{c}{a_0} \cos\left(\frac{2\pi(t - t_0)}{11{\cdot}13} + \epsilon_1 \right) \right\} - b_0 \left\{ 1 + \frac{d}{b_0} \cos\left(\frac{2\pi(t - t_0)}{11{\cdot}13} + \epsilon_2 \right) \right\} \sin^2 \phi \right] \cos \phi,$$

where a_0 and b_0 are constants, mean values of the variable a and b of relation (A) throughout the eleven-year cycle; c and d are constants, semi-amplitudes of the total observed variation of a and b; t the time (in years) from some fixed epoch t_0 in the 11·13-year cycle of sunspots; and ϵ_1 and ϵ_2 constants to adjust the phases of the variation of a and b to the chosen epoch. In the treatment of the observations it would seem to be desirable to deduce the values of a and b for the equatorial and polar regions by separate least-square solutions. The values roughly indicated are $a_0 = 2{\cdot}00$, $b_0 = 0{\cdot}400$, $c = 0{\cdot}6$, $d = 0{\cdot}4$. It would be rash to assign the relative values of ϵ_1 and ϵ_2 or to choose t_0 from data now available. If we try to form an idea of the cycle of changes in the solar circulation that appear to be indicated by the observational data of rotational velocity, the following may serve as a first attempt. In it I enter on the cycle at the epoch of minimum frequency of sunspots.

At sunspot minimum the vapours on the Sun at about latitude 50° lag behind those in contiguous zones of latitude with maximum retardation ($b = 0{\cdot}7$ or $0{\cdot}8$). The equatorial vapours are then moving with their lowest speed ($a = 1{\cdot}94$ or $1{\cdot}95$), but they are then beginning (with the disappearance of spots at the equator) to regain speed (possibly by reason of the tangential stresses between them and the effectively solid nucleus below them). The critical relative velocity between contiguous zones of latitude is reached somewhere below latitude 40° with outburst of spots in those latitudes (*e.g.* in 1901). The changes that follow are most tersely summarised in tabular form.

	Spots.	Velocities of Equatorial Region.	Velocities in Polar Regions 40° to 90°.	Prominences.
1900	At minimum			
1901	Burst out at Lat. 40°	$a' = 1{\cdot}95$ Low velocity	$b' = 0{\cdot}8$ Low velocities	Max. at 50° between 47° and 62°
1902	Lats. 27° to 18°	Increasing	$b' = 0{\cdot}7$ Increasing	,, 52 ,, 52 ,, 70
1903	Lats. 28° to 12°	Increasing	$b' = 0{\cdot}3$ Increasing	,, 52 ,, 52 ,, 70
1904	Lats. 30° to 5°	$a' = 2{\cdot}00$ Increasing	$b' = 0{\cdot}0$ Steady	,, 60 ,, 60 ,, 70
1905	Lats. 35° to 4° At maximum	$a' = 2{\cdot}05$ Increasing	$b' = 0{\cdot}1$ Falling	,, 68 ,, 68 ,, 80
1906	Lats. 27° to 3°	$a' = 2{\cdot}07$ Steady. High	$b = 0{\cdot}3$ Falling	,, 72 ,, 72 ,, 90
1907	Lats. 26° to 2°	..	..	Polar prominences die out at poles
1908	Lats. 23° to 2°	$a' = 2{\cdot}07$ Steady. High	$b' = 0{\cdot}7$ Falling	
1909	Lats. 17° to 2°	$a' = 2{\cdot}07$ (?) Falling	$b' = 0{\cdot}8$ Steady. Low	
1910	Lats. 16° to 2°	..		
1911	Lats. 11° to 1°	$a' = 2{\cdot}00$ Falling	$b = 0{\cdot}8$ Steady. Low	
1912	Lats. 21° to 9°	..	$b = 0{\cdot}8$ Steady. Low	
1913	At minimum, none near 0°	$a' = 1{\cdot}99$ Falling	$b' = 0{\cdot}8$ Steady. Low	
1914	Burst out at Lats. 32° to 19°, none at 0°	$a' = 1{\cdot}95$ Steady. Low	? Increasing	

The dark filamentous flocculi recorded on so many recent spectroheliograms have special interest in several ways. (*a*) They are regarded as prominences seen by absorption as dark regions on the brighter background of the disc of the Sun. If their pressure is low and their absorptive power is consequently only large for wave-lengths in the immediate vicinity of their monochromatic emission, their apparent ability to cut out the light from the background must be due to some property other than absorptive power. Can it be that they appear dark by reason of their diverting the light of the background by refracting it in various directions? (*b*) Their enormous length compared with their width is a striking feature, not exactly what one would expect from eruptive matter spreading in higher regions of the Sun's envelope. (*c*) Their capricious movements sometimes towards sunspots, sometimes in apparent disregard of sunspots or even in repulsion from spots, suggests that they must consist sometimes of negatively electrified ions, sometimes of positively electrified ions; and if such ions are organised in huge vortices, then their capricious movements would receive explanation partly as a result of a conflict between the actions of the general magnetic field of the Sun and of the powerful local fields discovered by Hale in sunspots. It is not easy to explain their long-continued persistence without the disruption that is to be expected in vortices of ions of the same sign of electrification. It would be interesting to know whether a spot is from the moment of its outburst always the seat of a magnetic field or only when one of these dark filaments has been drawn into it. If such a filament were really a vortex of definite electrical sign and in definite mechanical rotation, it would go a long way to remove one of the difficulties connected with the explanation of the origin of magnetic fields in the sunspots, viz. the separation of the ions of opposite signs in a purely mechanical vortex. The valuable studies of Deslandres, Hale, Evershed and Royds have shown us that these dark filaments may serve to indicate the directions of a circulation in the higher regions of the Sun's envelopes.

On a Proposal of a New Method of securing Spectroscopic Observations of the Rotation of the Sun. By H. F. Newall, F.R.S.

A sphere viewed from a distance looks like a circular disc. If the sphere is rotating about an axis, the position of the projected axis may be represented by a straight line drawn across the apparent disc in the appropriate position angle. A chord drawn on the disc parallel to the projected axis may be regarded as representing the edge-on-view of a small circle AC on the sphere cut by a plane whose central normal is perpendicular to the axis and also to the direction of view. It is easy to show that if the sphere were rotating like a rigid body about the actual axis and were viewed from a point so distant that the *variation* in the inclination of the line of sight to the axis may be neglected, then the motion in the line of sight of every point in the area of the small circle AC is the same, for any given inclination of the axis to the line of sight; and clearly the motion in the line of sight of every point in the area of a second small circle BD similarly situated on the other side of the axis of rotation has the same numerical value though of opposite sign.

It has occurred to me that it may be well to utilise a corollary of this property of a rotating rigid sphere in determining by spectroscopic observations the law of rotation of the Sun, which is known to be rotating according to a complex law different (at any rate on what we call the surface of the Sun) from the simple law of a rotating rigid sphere. We should then in a sense reduce the problem to a determination of the law of departure of solar rotation from rigid rotation.

Utilising the terms equator, latitude, longitude, and central meridian in their usual signification, we see that if two chords AC, BD are taken parallel to the rotation axis NS and equidistant from it on opposite sides, we may measure with the spectroscope the velocity-shifts for pairs of points P and Q equidistant from the equatorial diameter EW. Deducing (for the conditions of solar presentation obtaining at the time of observation) the latitude of the points P and Q and the longitude measured from the central meridian, we can obtain the solution of our problem in a way which allows us to study the law of rotation in each hemisphere of the Sun separately. For purposes of convenient description we may call these equal parallel chords (AC, BD) "the velocity chords."

According to the particular object of our research, we must choose the velocity chords suitably; *e.g.* for the purpose of search for change in rotational velocity in the spot-bearing zones between the equator and the latitudes $\pm 40^\circ$, we may take as velocity chords the chords passing from North to South through points on the limb midway between the equator and the axial points on the limb; or again, if we wish specially to study the rotational velocities in high latitudes, we must take the velocity chords much closer together.

For the sake of keeping the number of observations within reasonable limits, it is well to calculate beforehand the linear distances of P and Q from the equatorial diameter EW, for definite latitudes such as 0°, 10°, 20°, 30°, 40°, or 60°, 70°, 80°, 85°. In this way we shall accumulate

observations at definite latitudes, and so be in a position to detect possible variations of rotational velocity in a given latitude at different epochs in the sunspot cycle.

One great advantage, in cases where simultaneous observations of the two points P and Q are to be made by means of reflecting prisms, is that the prisms can be kept fixed in permanent positions, allowance being made in the calculations of latitude and longitude for the change in the apparent diameter of the Sun in the course of the year.

But the greatest advantage of the method lies in the ability of the observers to see if there are any obvious local disturbances at the points P and Q on the velocity chords, a matter of considerable doubt when observations are made close to the limb of the Sun.

The Variability of Nova Cygni, 1876 (*Schmidt*). By E. E. Barnard.

In *Monthly Notices* for 1902 March (62, 405) I have given a brief history of Schmidt's Nova Cygni of 1876, with observations of its position and brightness. A second paper on the same subject appeared in *Monthly Notices* for 1912 April (72, 525). These observations seemed to show that the star was then variable. Later observations confirmed this variability. Attention was called to this fact in *Astronomical Journal* for 1919 February 7 (32, 6).

I have made special observations for many years to determine the variability of the nova. These show that the star varies through about one magnitude. There seems to be some evidence roughly of a period of about forty-three days, but this is somewhat uncertain. To aid me in this work, Professor Seares, of Mount Wilson, very kindly determined the photo-visual magnitudes of my comparison stars. With these I have determined the magnitude of the nova at my various observations of it in the past twenty years. From an inspection of these observations its variability is readily shown. The following table contains Professor Seares' magnitudes of the comparison stars. He says that these results depend on three pairs of comparison plates with the North Pole in 1920 November 16 and 17.

Professor Seares' Magnitudes of the Comparison Stars.

Star.	Photographic Mag.	Photovisual Mag.	Colour-Index.	Star.	Photographic Mag.	Photovisual Mag.	Colour-Index.
Nova	14·80	14·73	+0·07	51	15·97	14·71	+1·26
37	16·25			54	14·49	12·84	1·65
41	15·21	13·98	1·23	57	14·65	13·79	0·86
42	16·28	15·02	1·26	60	13·26	12·53	0·73
45	15·87	14·73	1·14	62	16·50		
47	13·95	13·29	0·66	68	15·13	14·44	0·69
49	15·53	14·85	0·68	70	15·11	14·76	0·35
50	16·73	(15·3)	(1·4)	73	14·97	14·06	0·91

Professor Seares says: "The exposures with the filter were not long enough to show stars 37 and 62; 42 and 51 appear on two plates only, and the photo-visual magnitudes are not very reliable; 50 is faintly visible on a single plate. The value given for the photo-visual magnitude is a rough estimate."

The numbers are those of the Copeland and Lohse list in *Copernicus II.* (1882), pp. 117–119. I have determined the approximate magnitude of No. 62 from comparisons with No. 50. These observations make it very slightly less (about 0·03 mag.) than 50, so that it may be assumed to equal 50, or of magnitude 15·3. The times are $6^h\ 0^m$ slow of G.M.T.

Date.	C.S.T. h m	Mag.	No. of Comps.
1901 Nov. 4	9 10	14·65	2
5	10 0	14·71	1
12	7 0	14·69	2
18	11 0	14·71	1
19	7 0	15·21	1
Dec. 4	7 20	14·87	2
17	6 35	14·90	2
22	6 20	15·20	3
29	7 10	14·75	4
30	6 5	14·81	2
1902 Jan. 2	6 40	15·07	3
5	7 35	15·17	3
6	6 30	14·95	4
7	6 10	14·93	6
10	7 15	14·90	2
11	7 15	15·00	2
12	6 20	14·97	3
13	6 20	14·99	4
14	6 35	14·92	2
18	6 30	14·92	3
19	6 45	<14·71	1
26	6 30	14·90	2
27	6 20	15·05	2
31	6 15	14·90	2
1911 July 16	11 45	15·21	1
21	11 30	14·87	3
25	13 55	14·51	1
30	15 0	14·46	1
Aug. 20	13 30	14·40	2
1911 Aug. 29	12 0	14·86	2
Sept. 1	12 0	14·69	3
3	14 30	14·91	1
Oct. 29	8 30	14·71	1
1917 July 28	12 0	15·10	2
1918 Sept. 28	12 30	14·61	3
Oct. 3	12 0	14·36	5
5	8 20	14·98	7
8	11 15	14·57	4
12	9 0	14·72	4
29	11 10	14·77	4
Nov. 2	9 20	14·46	1
5	10 0	14·67	4
12	7 20	14·66	4
23	7 55	14·35	4
26	7 25	14·32	10
30	7 15	14·49	6
Dec. 3	10 20	14·72	4
7	7 15	14·58	4
1919 Jan. 18	7 30	14·74	6
Feb. 4	7 15	14·76	3
July 5	13 0	14·33	2
8	14 0	14·41	1
10	13 50	14·30	2
17	13 30	14·38	2
19	13 35	14·51	1
22	12 0	14·51	3
Aug. 5	10 20	15·34	6
	13 35	15·05	2

1901 Nov. 19; 1911 July 16, Aug. 20; 1918 Oct. 5, Nov. 2, Nov. 5, uncertain.
1902 Jan. 19, not seen. 1911 Sept. 3, sky poor.

Date.	C.S.T. h m	Mag.	No. of Comps.
1919 Aug. 9	11 45	15·08	6
19	11 10	14·02	4
21	12 20	14·33	4
23	10 55	14·25	4
25	12 35	14·32	3
28	12 45	14·41	3
Sept. 11	11 5	14·30	4
16	10 40	14·82	4
20	9 0	14·47	3
23	10 20	14·60	4
27	12 15	14·54	5
Oct. 2	11 0	14·52	5
7	8 50	14·55	4
11	10 15	14·62	2
14	10 20	14·61	5
16	11 10	14·96	4
18	9 35	14·87	5
21	8 45	14·89	4
28	8 10	14·84	4
Nov. 1	9 10	14·60	2
11	8 5	14·57	4
15	8 50	14·60	4
18	9 33	14·81	4
22	8 20	14·65	4
Dec. 2	7 30	14·50	4
1920 Oct. 5	10 5	14·22	10
1920 Oct. 7	11 30	14·32	10
9	9 30	14·51	12
12	9 50	14·55	10
19	11 30	14·68	6
21	11 25	14·99	3
28	11 0	14·51	1
30	7 45	14·85	13
1921 June 28	12 0	14·78	6
July 2	12 30	14·92	5
7	13 10	14·82	7
9	12 15	14·81	7
12	12 55	14·97	7
30	11 50	15·01	7
Aug. 2	10 20	14·86	7
9	11 0	14·98	6
18	11 45	14·55	4
20	10 55	14·63	10
25	12 30	14·32	6
Sept. 1	12 15	14·36	7
6	9 40	14·32	7
8	13 10	14·28	8
22	12 0	15·09	9
27	7 20	15·27	7
29	8 20	15·28	7
Oct. 4	8 25	15·44	7

Though these observations are given to the hundredth of a magnitude, it is not certain that they are reliable to the first decimal place.

The changes of brightness in the nova are sometimes rather sudden. This is well shown in the observations of 1921 September, when in a few days' time the star suddenly dropped from 14·3 magnitude to below the fifteenth. Unfortunately, though the nova was observed on every possible occasion, cloudy weather prevented a close watch of this minimum phase.

Observations will be continued, and the period and light-curve determined, if there is really a definite period to its fluctuations. The nova is rather difficult to observe, because its image is not perfectly stellar. The focus is slightly greater than that for a normal star, about 2 mm. (See *M.N.*, 72, for 1912 April, bottom of page 527.) Thus is definitely added one more to the variable novæ.

The following additional measures were made of the nova and star No. 60:—

Nova and 60.

1917·573	July 28	70°·58	52″·93
1918·904	Nov. 26	70·41	53·10
1918·915	,, 30	70·14	52·89
1919·739	Sept. 27	70·14	53·07
1918·782		70·32	53·00

For previous measures, see *M.N.*, 62, 408, and 72, 526.

Yerkes Observatory,
Williams Bay, Wisconsin:
1921 *October* 7.

[*Note by H. H. Turner, added* 1921 *October* 29.—Barnard's suggestion of a period near 43 days is of importance in connection with the suggested variation of other novæ. The result of a few calculations may be noted. There is a well-observed run of nearly 1200 days, from 1918 September 28 to 1921 October 4. Consecutive divisors of 1200 near 30 are—

Divisor	27	28	29	30	31	32	33
	d	d	d	d	d	d	d
Period	44·5	42·9	41·4	40·0	38·7	37·5	36·4
Adopt	44·5	43·0	41·5	40·0	38·7	37·5	36·5
$2 \sin \theta_0$	−16	+14	+32	−12	+ 8	+29	−1
$2 \cos \theta_0$	+ 7	− 6	−20	− 4	+23	+ 9	−2
I.	305	232	1424	160	593	922	5

The figures show a rough Fourier analysis of these observations. The second line shows the period corresponding to the divisor above it, the third line a slightly modified period for easier calculation; the fourth and fifth lines give twice the deduced coefficients of $\sin \theta$ and $\cos \theta$ for the adopted period, and the sixth the sum of squares of 4 and 5—a measure of the intensity of the assumed periodicity. It will be seen that the most likely value of the period is 41·5 days, with a possible companion variation at 38 days, which would "interfere" with the former in

$$11 \times 41{\cdot}5 = 12 \times 38 = 456 \text{ days.}$$

The coefficient of the 41·5-day period is 0·20 mag., and of the 38-day 0·15 mag.

The earlier observations are available for possible confirmation of this variation: but the observations from 1901 November 4 to 1902 January 31, covering two periods, do not show a large coefficient—0·10 mag. at most,—and the intermediate observations from 1911 July 16 to 1917

July 28 are all confined to one half of the cycle. Hence it is scarcely possible to assign an accurate period at present, but the best value seems to be 41·5 days. In any case, this cannot account for the whole of the variation, which is often abrupt.]

Additional Observations reported 1921 *December* 12.

Further observations of the nova have been obtained in the interval of waiting for the proof, and are here added. The bad weather indicated in the next to the last paragraph of my paper, continued and still continues, so that observations of the rise from minimum, which is not yet complete, have been badly interfered with.

Date.	C.S.T.	J.D.	Mag.	No. of Comps.
1921 Oct. 4	8 25	22967	15·44	(7)
13	10 50	22976	15·18	(4)
15	9 40	22978	14·85	(2)
20	11 10	22983	15·00	(6)
22	10 5	22985	15·11	(7)
Nov. 1	8 17	22995	14·71	(14)
24	11 20	23018	14·84	(8)
29	7 40	23023	14·83	(8)
Dec. 6	6 50	23030	14·81	(17)
10	6 50	23034	14·68	(7)

1921 Oct. 13, bad seeing, somewhat uncertain; Oct. 15, uncertain, moonlight and bad seeing; Nov. 1, difficult from bad seeing; Dec. 6, seeing good, but strong moonlight; Dec. 10, nearly full moon, but fairly well seen.

[Taking the maxima of the 41·5-day period as falling at J.D. $22005 + 41{\cdot}5n$, when $n = 24$ and $n = 25$ we get maxima at 23001 and 23042·5, which seems to accord fairly well with the above recent observations.—H. H. T.]